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SOME STATISTICAL AND DYNAMICAL ASPECTS OF THE FISSION THEORY

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ABSTRACT

I. *Statistical.*—The assumption that widely separated eclipsing binaries have been formed from close pairs by the process of contraction requires that the initial densities of the present wide pairs were systematically lower than those of the present close pairs.

The questions of a reduction in the mass of binary stars, and of the effect of the limited time scale introduced by the idea of an expanding universe, are discussed.

Present indications are that the stars are not picking up enough interstellar matter to balance the loss of mass by radiation, but the subject is considered an open one.

II. *Dynamical.*—Application of the formula $\rho < C/P^2$, where P is the period of a binary formed by fission, C is a known function of its mass ratio, and ρ is the density of the "liquid core" of the parent star, to the β Lyrae variable W Crucis and to Capella shows that the densities of the cores of their parent-stars were of the order of 10^{-6} gm/cm³. The implication is that they were *gaseous* and not liquid. A gaseous star is not subject to fission if the ratio of its central density to the mean is greater than 2.7, and this ratio is exceeded in most of the stellar models heretofore presented.

The law of internal density of a model by Rosseland is such that it might become either lens-shaped or ellipsoidal upon contracting. If it becomes lens-shaped, then an *immediate* increase in the mean density of at least 40 must occur during disruption, if a binary of mass-ratio unity is to result.

The first part of this paper considers statistically certain problems arising in a study of the evolution of binary stars. The second part discusses the possibility, from a dynamical point of view, of the formation of the binaries W Crucis and Capella by fission.

I

1. In a previous paper[1] the densities, ρ, of the components of 125 eclipsing binaries were plotted against their relative separations, σ,

[1] *Astrophysical Journal*, **75**, 69, 1932.

on page [79]. (We shall use square brackets to denote a reference to the previous paper.) The relative separation was defined to be the ratio of the distance between the surfaces to the distance between centers.

Had ρ been the same, on the average, for all the binaries initially, that is, when $\sigma = 0$ and the components were in contact, then the densities of those binaries whose relative separations are large should be systematically higher than of those whose separations are small, if contraction had occurred. It was found, however, that the average value of ρ was the same for all σ-classes (see Fig. [1] and Table [IV]), and the data were interpreted as showing that "the components of close binaries are not contracting." Since the initial densities are not known, it follows, of course, that the data just given do not prove that such contraction might not have occurred.

It might well be that the stars which had a very low initial density have contracted more than the others, so that while their values of σ are now large, their values of ρ are about the same as those which have small σ. Hence, Professor H. N. Russell suggested that I determine ρ_0, the initial density, for each of the 250 components, assuming that they contracted from a state in which, originally, the components were in contact. These computations are carried out by means of equations [4] and [5].

The values of ρ_0 were determined for two different cases. In one it was assumed that tidal forces do not affect the absolute separation. In the other case it was assumed that they do so by the maximum amount possible. The average value of $\log_{10} \rho_0$ for each of these assumptions, and for each of ten σ-classes is given in Table I. Column 1 gives the upper limit of σ for each class. In Figure 1 the individual values of $\log \rho_0$ for no tidal forces were plotted against σ.

These results indicate that if the present large values of σ for some of the binaries are due to contraction, then these binaries had very much lower densities, initially, than those for which σ is now near zero. Professor Russell has suggested that some of this difference may be due to observational selection.

We have already mentioned the fact that according to Table [IV] the arithmetic average of ρ for each σ-class is about the same. A more accurate picture of the situation is obtained, however, if the

detailed distribution of the 250 components is given, as in Table II. For this purpose the binaries were separated into three large classes, giving us close, medium, and wide pairs. In class I, σ runs from 0

TABLE I

σ	No Tide	Tide
0.1	8.53-10	8.60-10
0.2	8.60	8.81
0.3	8.79	9.16
0.4	8.28	8.75
0.5	7.94	8.54
0.6	7.87	8.50
0.7	7.74	8.44
0.8	6.46	7.86
0.9	6.19	7.05
1.0	5.56	6.43

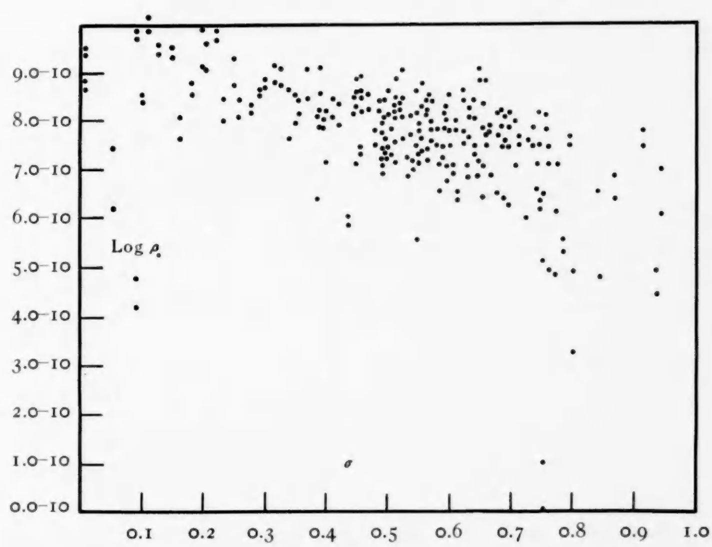

FIG. 1.—The logarithm of the initial density, calculated on the assumption that contraction has occurred, plotted against the present relative separation. The densities are on the solar standard.

to 0.3; in class II, from 0.3 to 0.6; and in class III, from 0.6 to 1.0. As is shown in column 1, six classifications of density have been made. The last three columns give the percentage distribution ac-

cording to the density for each σ-class separately. The other columns give the actual numbers. A glance at the percentage columns indicates that there is no appreciable difference in density distribution for the close, medium, and wide pairs investigated.

The distribution with respect to spectral type of 123 of the 125 systems whose types are known has been similarly determined, and the results entered in Table III. The entries in the fourth row are for dwarfs, and those in the fifth for giants. Here again the distribu-

TABLE II

ρ	I	II	III	I	II	III
$\geqq 1$	5	0	3	12%	0%	3%
.3 1	13	16	22	31	13	25
.1 .3	7	35	26	17	29	30
.01 .1	10	60	21	24	50	24
.001 .01	4	6	11	9	5	12
$<$ .001	3	3	5	7	3	6

TABLE III

	I	II	III	I	II	III
B	5	16	3	24%	27%	7%
A	9	39	33	43	65	79
F	5	3	3	24	5	7
d (G, K, M)	1	0	1	5	0	2
g (G, K, M)	1	2	2	5	3	4

tion is seen to be the same for close, medium, and wide pairs. Since contraction probably changes the spectral type of a star, it follows that the wide pairs did not, if they have been formed by contraction, originally have the same spectral distribution as the present close pairs.

We are now in a position to sum up the case. Tables II and III indicate that the distribution of the observed eclipsing binaries with respect to density and with respect to spectral type is independent of the relative separation. These results can be explained by assuming that either (*a*) the stars have not contracted greatly during the period of their past history within which we can reason about them; or (*b*) if they have so contracted, the densities at which they divided into

two components were systematically much smaller for the present wide pairs than for the present close pairs. To determine which possibility is correct requires that we determine the past history of close and of wide pairs—two groups of stars at present physically alike. This is something which cannot be done by statistical methods, and therefore one must form an opinion from other considerations.

To explain the fact that wide pairs do not have higher densities than close pairs, Professor Russell states in a letter sent to me:

> There is strong evidence that the densities of stars belonging to the main sequence, and of any particular spectral type, are confined within a narrow range. The reason for this is not certainly demonstrated, but it appears probable that it depends on some general physical relation, independent of the existence of a binary companion. For example, it may be necessary that the central temperature shall approach a certain value in order to provide a sufficient supply of heat from sub-atomic sources. . . . . The point is that if a star of given mass must reach approximately a given density, in order to become stabilized on the main sequence, this will happen at this density no matter whether it is double or single, and no matter how far off its companion may happen to be.
>
> The conclusion which I think therefore may be drawn from the data is that the conditions, whatever they may be, which cause stars of a given mass to show a strong statistical preference for certain radii and surface temperatures are nearly independent of past fission (if this occurred), and of the presence of companions. . . . .
>
> I have made this argument for main sequence stars of a particular mass and spectral class. It would apply equally to stars of any other mass, with the appropriate spectral class, and to giants, instead of main sequence stars.

If Professor Russell's hypotheses are correct, then we have an explanation of why the average density is independent of separation. However, the range in density for any of the σ-classes of Table II is quite large; and yet the distribution according to density is the same for all three classes. For a group of binaries to start with widely differing densities, and then to contract by differing amounts so that finally the foregoing condition holds, seems, to the writer, somewhat improbable. Hence, it is his personal opinion that eclipsing binaries with large relative separation have not been formed from those of small separation by a process of contraction.

2. In a discussion of the frequency distribution with respect to σ of eclipsing binaries it was found that the number decreases sharply

when σ was less than 0.3 (see Fig. [2]). On page [84] it was stated: "Hence, the conclusion that might be inferred from a study of Figure 2 and Table IV is that the components of close binaries are being drawn together, systems in which one component is much denser than the other being disrupted first." The conclusion just quoted was one suggested by the data, but I had not meant to imply that it was the only one consistent with the data.

3. In section [V] the relation between the periods and masses of spectroscopic binaries was discussed. Had their initial periods and masses been alike, then, if the periods had since been increased because of a decrease of mass, we should expect M to decrease with P. It was found, however, that the average value of M was the same for all period groups—about 3 ($\odot = 1$). The conclusion drawn from the data was that long-period spectroscopic binaries had not been formed from short-period binaries because of a decrease of mass.

We have here the same kind of difficulty which appeared when discussing the densities of binaries, namely, the question of the initial masses of the binaries in the different period groups. If we assume that the present independence of P and M is due to the fact that binaries for which P is large started out with large masses, then, for example, stars for which $P = 100$ days had $M = 30$ just after fission. Binaries of such high masses are known, but are exceedingly rare. Whether or not there were a sufficient number of these in the past capable of developing into the large number of long-period spectroscopic binaries now known is something we do not know.

Professor Russell calls attention to the fact that here again observational selection may have an important bearing on the problem. There will be a tendency to observe the luminous giants of large mass in preference to the faint dwarfs of low mass. It may be, then, that the actual average M for the period groups of large P is smaller than that deduced from observation. However, if selection is to account entirely for the observed independence of P and M, then the true average value of M for binaries of 100-day period should be about 0.3 instead of the observed 3.0.

E. Öpik[2] has made a study of the relation between the luminosities and separation of the components of visual binaries, in a manner

[2] *Popular Astronomy*, **41**, 71, 1933; *Tartu Observatory Publications*, **25**, 6, 1924.

free from observational selection. His conclusion is also that the masses of these binaries have not been diminished, appreciably, since their formation.

4. A question which has recently arisen in the study of the evolution of binary stars is that of the time scale introduced by the concept of an expanding universe. If the red shift in the spectral lines of certain spiral nebulae is interpreted as a real velocity, then, as is well known, our galaxy has existed in its present form only about 10^{10} years. The periods of time required to make significant changes in the masses and orbits of binary stars are so large that one may well say that on the basis of a time scale of 10^{10} years there has been no evolution of binary stars since the galaxy took its present form. During this period of time, for example, the sun would lose 0.1 per cent of its mass if its present rate of radiation were constant. The more massive stars will lose somewhat more; a giant star ten times as massive as the sun might lose 10 per cent of its mass. A time scale of 10^{10} years, therefore, would be in agreement with the foregoing results of Öpik and the writer. In fact, because of his results, Öpik adopts this time scale as the correct one for the age of the galaxy. The writer, however, does not attempt at this time to express an opinion of the interpretation of the red shift and the consequences resulting therefrom.

5. The failure to detect changes in the masses of stars, from the statistical viewpoint of the last section, may be due, not to a short time scale, but to the fact that the loss by radiation is balanced, on the average, by the picking-up of interstellar matter. A cyclical process to account for the latter was suggested in 1918 by Professor W. D. MacMillan.[3] The difficulty in this theory is that the density, ρ, of interstellar matter appears, at the present time, to be of the order of 10^{-23} gm/cm^3—too small to balance the loss by radiation.

The rate at which matter is picked up varies inversely as V, the relative velocity of the star and the medium. If V were 10 km/sec., then ρ would have to be 7.2×10^{-20} if the mass of the sun were to be constant. For the same velocity ρ would have to be 1.7×10^{-17} for the mass of the binary H.D. 1337 to be constant, and 7×10^{-21} for Krüger 60. The total mass and emission of energy of H.D. 1337 are,

[3] *Astrophysical Journal*, **48**, 35.

respectively, 70 and 700,000 times that of the sun, and for Krüger 60 these quantities are 0.5 and 0.021.

It may be that a star picks up matter in appreciable amounts only when passing through a nebula with a low relative velocity. The time of passage through a nebula varies inversely as V, and so does the rate at which matter is picked up, so that the total amount picked up varies inversely as V^2.

The "General Catalogue of Radial Velocities"[4] by J. H. Moore lists ten O- and B-type stars in the Orion nebula whose radial velocities relative to it are about 3 km/sec. (5 km/sec. are allowed for the K-term of the stars). Let us assume that the diameter of the nebula is 5 light-years and that its density is 10^{-17} gm/cm^3. Then if the sun passed through it with a velocity of 3 km/sec., enough matter could be picked up to last 2×10^8 years. Similarly, H.L. 1337 could pick up enough for 10^6 years, and Krüger 60 enough for 2×10^9 years. It is assumed here that radiation pressure does not prevent the infall of matter, as was shown by Russell[5] to be possible under certain circumstances.

It is a familiar fact, however, that the mass of the Orion nebula would be so large, for the assumed size and density, as to affect the velocities of neighboring stars appreciably. The parabolic velocity at the surface would be 58 km/sec., and this number times $(3/2)^{\frac{1}{2}}$ at the center if the nebula were homogeneous. A short calculation shows that the stars in the nebula whose relative velocities are 3 km/sec. are permanently associated with it if its density exceeds 10^{-21} gm/cm^3.

It seems, thus, as though the loss of mass of stars by radiation is not being compensated for by the picking-up of interstellar matter. It is felt, however, that a more definite idea of the extent, masses, and densities of the gaseous nebulae is required before we can state so definitely.

II

6. We shall now discuss some dynamical phases of the process of fission. An outline of the fission theory was given in the previous

[4] *Publications of the Lick Observatory*, **18**, 1932.

[5] *Astrophysical Journal*, **69**, 49, 1929.

paper on page [10], but it is advisable to present here some of the details given there and some additional ones also.

If a rotating, homogeneous, incompressible liquid body contracts, its figure of equilibrium goes through a certain sequence. The first member is a sphere. Then come spheroids of revolutions, called "Maclaurin spheroids." The first member of the spheroids is the sphere, and the last member—we shall call it the "last Maclaurin spheroid"— has an eccentricity in the plane of a meridian of 0.8127. After the spheroids come the ellipsoids of Jacobi, with three unequal axes, the first member of this series being the last Maclaurin spheroid. According to the computations of Sir George Darwin, the last stable Jacobi ellipsoid has the following properties: if r is its mean radius with respect to volume, then its semi-axes are: $a = 1.8858r$, $b = 0.8150r$, $c = 0.6507r$. Its moment of momentum is equal to 0.3898 $M^{\frac{3}{2}} r^{\frac{1}{2}} k$, M being its mass and k^2 the gravitational constant.

The sequence of figures just mentioned are all stable. If a body should contract beyond the last Jacobi ellipsoid, the figure of equilibrium would be a Poincaré pear-shaped body. But this is unstable, and the star would be subject to a cataclysm of unknown nature. According to the fission theory, a spectroscopic binary will result in which the components will be nearly in contact and whose orbital eccentricity will be near zero. The liquid sequence, then, begins with a sphere and ends with the last Jacobi ellipsoid.

For a contracting body to go exactly through the foregoing sequence it is necessary that it be liquid, homogeneous, and incompressible. If it is a compressible, gaseous body, then its sequence of figures of equilibrium is likely to be far different. This sequence can be easily determined if the body has a uniform angular velocity, and has enough of its mass situated at its center so that the gravitational potential for points on its surface varies inversely as the distance to the center. We shall refer to the sequence for a body having this property as the gaseous sequence.

For zero angular velocity the figure of equilibrium is a sphere. For slow rotations it is an oblate spheroid. If the body contracts the angular velocity increases, and there is a tendency to form a lens-shaped figure with a sharp edge at the Equator. Finally a sharp edge is attained, and at this point the polar diameter of the figure is equal

to two-thirds its equatorial. The attraction by the main part of the body for the particles in the sharp edge is exactly balanced by the centrifugal force due to rotation. If the body contracts after having reached the end of the sequence, these particles are left behind to revolve as minute satellites around the main mass. Thus, if contraction continued, matter would be left behind in the form of a flat ring, as in the rings of Saturn, while the contracting body would always have the figure of the last member of the gaseous sequence. What we have stated here is nothing more, of course, than the Laplacian theory of the formation of the solar system.

Sir J. H. Jeans[6] has investigated the equilibrium figures of rotating gaseous bodies assumed to be in adiabatic equilibrium for the purpose of determining the degree of central condensation required in order that they may shed matter at the Equator upon contraction. He finds that this will happen if κ, the ratio of the specific heats of the gas, is less than 2.2. Since κ is related to the familiar polytropic index n by the relation $n=1/(\kappa-1)$, it follows that n must be greater than 0.8. From the tables of R. Emden[7] I find, by interpolation, that the central density of the polytrope $n=0.8$ is 2.7 times the mean. It appears, therefore, that a gaseous star whose central density is at least 2.7 times the mean density will not form a binary star by the process of fission.

To account for the formation of binary stars by fission, then, Jeans supposes that the interiors of stars are in a liquid (incompressible) or semiliquid state, surrounded by an envelope of negligible mass. The liquid core of the star is supposed to go through the process of fission within the star, unaffected by the presence of the envelope. Jeans's theory of stellar structure requires, in fact, that all stars have liquid cores. He says:

> To sum up, we have found that considerations of stability demand that all stars should be in a state in which the deviations from the gas laws are appreciable, while actually the majority of them are found to be in states in which these deviations are so large that their central regions may properly be described as in the liquid state.[8]

From considerations of the energy and the moment of momentum of a rotating, homogeneous, liquid body before and after fission,

[6] *Astronomy and Cosmogony*, p. 250, 1928.

[7] *Gaskugeln*, p. 78, 1907.

[8] *Op. cit.*, p. 160.

F. R. Moulton[9] and Russell[10] have obtained limits on the amounts by which the period and the major axis of a binary may be increased as a result of mutual tidal forces. In his paper Moulton also established, essentially, the following result:

If a rotating, homogeneous, liquid body, of density ρ_J at the time of fission, divides and forms a binary star whose orbital elements are not changed thereafter by any forces other than those due to mutual tides, then

$$\rho_J < \frac{C}{P^2},$$

where P is the period of the binary and

$$C = \frac{.39^6 \, 3\pi \, (1+c)^{12}}{(1-e^2)^3 k^2 \, c^6},$$

c being the mass ratio of the binary, e its orbital eccentricity, and k^2 the gravitational constant.

For the binaries which we shall consider, e is so small that we may put it equal to zero. If the units are so chosen that ρ_J is given in gm/cm^3 when P is given in days, then $C = 0.27$ when $c = 1$; 0.55 when $c = 0.5$; and 4.05 when $c = 0.25$.

If Jeans's model, consisting of a liquid nucleus surrounded by an atmosphere of negligible mass, is followed in nature, then we can use this formula to obtain upper limits for the density of the liquid cores of the parent-stars of binaries supposed to have been formed by fission. This has been done for several such systems and the results have been entered in Table IV. With the exception of Capella they are all eclipsing binaries. The quantity $(r_1 + r_2)/a$ gives the ratio of the sum of the radii of the components to the orbital major semi-axis. The fourth column gives ρ_J, the density of the liquid core of the parent-star when its figure of equilibrium was that of the last stable Jacobian ellipsoid. The quantity ρ_M is the density when its figure was that of the last Maclaurin spheroid, and the other columns give the density when (and if) its figure was a spheroid of eccentricity equal to the subscript of ρ. In computing these values it was as-

[9] *Carnegie Institution of Washington Publications*, No. 107, 150, 1909.

[10] *Astrophysical Journal*, **31**, 185, 1910.

sumed that $c=1$. The values given should be multiplied by 2 if $c=0.5$, and by 15 if $c=0.25$.

The mass ratio of Capella is practically unity. The mass ratio of W Crucis cannot be directly determined, but the photometric data indicate a luminosity ratio of 9, and by applying the mass-luminosity law we derive a mass ratio of 0.4. These two binaries are mentioned by Jeans[11] as having been formed by fission. W Crucis has all the characteristics of one recently formed; its orbital eccentricity is zero, and the two components are nearly in contact, the sum of the radii of the components being 0.95 of the orbital major semi-axis. Table IV shows that the density of the "liquid core" was of the order of 10^{-6} gm/cm^3, a density one-thousandth that of air, before fission.

TABLE IV

Name	P	$\frac{r_1+r_2}{a}$	ρ_J	ρ_M	$\rho_{.5}$	$\rho_{.25}$
ϵ Aurigae	9905^d	0.3	2.7×10^{-9}	6.1×10^{-10}	6.1×10^{-12}	6.1×10^{-14}
W Crucis	199.	0.95	6.8×10^{-6}	1.5×10^{-6}	1.5×10^{-8}	1.5×10^{-10}
Capella	104.	0.2	2.5×10^{-5}	5.6×10^{-6}	5.6×10^{-8}	5.6×10^{-10}
SX Cassiopeiae	36.6	0.25	2×10^{-4}	4.5×10^{-5}	4.5×10^{-7}	4.5×10^{-9}
β Lyrae	12.9	0.95	1.6×10^{-3}	3.6×10^{-4}	3.6×10^{-6}	3.6×10^{-8}
X Carinae	1.08	1.0	2.4×10^{-1}	5.4×10^{-2}	5.4×10^{-4}	5.4×10^{-6}

The density was even less when the core was in the spheroidal form, but we do not know what its initial equilibrium figure was.

It is hard to believe that matter at a density of 10^{-6} could be incompressible or even mildly so. It appears more probable that at such densities it would be in the form of a compressible gas subject to Boyle's law. If this is so, then the parent-stars of W Crucis and Capella, having the central condensation assumed by Jeans, should have contracted according to the gaseous sequence, and not the liquid. Thus it appears improbable that these giant eclipsing binaries have been formed from single stars by the process of fission in the manner suggested by Jeans.

Let us, however, consider some other stellar models. We have previously noted that a compressible star whose interior density varies in a manner similar to that of a polytrope for which $n>0.8$

[11] *Op. cit.*, pp. 176, 280.

(corresponding to a central density greater than 2.7 times the mean) will shed matter at the Equator upon contraction, but is not subject to fission. In most of the stellar models which have thus far been proposed, the law of density is similar to that of polytropes for which n is considerably greater than 0.8; e.g., in Sir A. S. Eddington's model, $n=3$, and in E. A. Milne's white dwarf model, $n=1.5$. The ratios of the central to the mean density for these models are 54 and 6, respectively. It follows, therefore, that if gaseous stars are actually similar to these models with regard to the density distribution, then they are not subject to fission, and that the giant binaries W Crucis and Capella have not been so formed.

We shall consider one more model, that due to S. Rosseland.[12] It is based on the assumption that the rate of generation of energy at a given point within the star is dependent on the density and temperature there. A peculiar property of this model is that the maximum density does not occur at the center of the star. G. Steensholt[13] had solved Rosseland's equations by the method of mechanical quadrature for several different cases. From the diagrams in his paper it appears that for these models the density reaches a maximum of about 1.5 or 2 times the central density about midway out from the center, and then drops to zero by the time the surface is reached. Its central density is 1.1 times the mean, and its maximum density is 2.2 times the mean in the second case mentioned.

What sequence of figures does such a star go through as it contracts? The ratio of its central density to its mean is less than that for the polytrope $n=0.8$. The sequence, however, is determined, not by this ratio, but by the fraction of the star's mass in its central region. Consider a star based on Rosseland's model and one similar to the polytrope $n=0.8$. Let each star have a mass equal to unity and a radius equal to unity. Let m be the mass of that part of the star lying within a sphere of radius r and concentric with the star. Then, by interpolation of Emden's tables and calculations based on the diagram of Steensholt, I derived the results tabulated. The val-

r	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
m ($n=.8$)	.003	.023	.068	.15	.28	.43	.60	.76	.90	1.0
m (Ros.)	.001	.011	.043	.12	.24	.46	.63	.80	.94	1.0

[12] *Zeitschrift für Astrophysik*, **4**, 255, 1932.

[13] *Ibid.*, **5**, 140, 1932.

ues of m for the two models are so nearly alike that it seems as if the Rosseland model may follow either the gaseous or the liquid sequence upon contraction.

Let us suppose, first, that it becomes lens-shaped, and also that because of its peculiar law of internal density it is subject to disruption, a binary being formed thereby. The moment of momentum, **M**, of the lens-shaped figure, if it rotates as a rigid body, is $\alpha\ M\ r^2\ \omega$, where M is its mass, r its mean radius, ω its angular velocity, and α its radius of gyration divided by r^2. For a homogeneous sphere $\alpha = 0.4$, and for the Rosseland model I find, from computations based on Steensholt's diagrams, that $\alpha = 0.3$.

The angular momentum of a rotating body may also be written as follows:

$$\mathbf{M} = AM^{\frac{3}{2}}\, r^{\frac{1}{2}}\, k\ , \tag{1}$$

k^2 being the gravitational constant and A being a dimensionless number. As has been previously noted, $A = .3898$ for the last Jacobian ellipsoid in the homogeneous series. According to Jeans,[14] $\omega^2/2\pi k^2 \rho \leqq .36075$ for the lens-shaped figure, ρ being its mean density. By means of this inequality I find that $A \leqq .221$ for this figure.

Let the binary resulting from the star's disruption rotate as a rigid body. The moment of momentum of the binary, equal to that of the parent-star, may be written as follows:

$$\mathbf{M} = BM^{\frac{3}{2}}\, R^{\frac{1}{2}}\, k\ , \tag{2}$$

where M is the total mass, R is the radius of a sphere whose volume equals that of the sum of the components, and B is a known function of the mass ratio, the ratio of the radii, the relative separation, and α.

As before, let $\alpha = 0.3$. If the ratio of the radii is unity, then $B \geqq .410$ when the mass ratio $c = 1$; $B \geqq .374$ when $c = 0.5$; and $B \geqq .296$ when $c = 0.25$. Within the range just given for c it is found that B changes but little even when the radii are far from equality. The minimum values of B occur when the components are in contact.

The ratio of the mean density of the binary formed by fission to

[14] *Op. cit.*, p. 255.

the mean density of the parent-star is equal to r^3/R^3, and by comparing equations (1) and (2) we find that

$$\left(\frac{r}{R}\right)^3 = \left(\frac{B}{A}\right)^6 .$$

We have seen that $A \leqq .221$ for the Rosseland model when it is lens-shaped. Values of B have just been given. Thus, it is found that if a binary star is to be formed under the assumptions made, then the mean density must increase immediately, during fission, by a factor of at least 41 if the binary has a mass ratio of 1.0, by at least 24 if its mass ratio is 0.5, and by at least 6 if the mass ratio is 0.25. It is a peculiar fact that the value of A is just about large enough for a homogeneous body having the shape of the last Jacobian ellipsoid, so that a binary of mass-ratio unity could be formed from it without an increase in density.

In order that a body may form a binary by fission without the necessity of a large immediate increase in density, it is necessary that its figure be one with a high moment of inertia so that it may have a large angular momentum and a small angular velocity simultaneously. The moment of inertia of the last stable homogeneous ellipsoid, for example, is 2.1 times that of a homogeneous sphere of equal mass and volume. The Rosseland model, as we have seen, may become ellipsoidal, but its moment of inertia is less than that of a homogeneous body of the same mass and shape. This is because there is a concentration of mass toward the center, and because the layers of equal density tend toward sphericity as the center is approached. Hence, it might be necessary that a star based on this model must become even more elongated than the last Jacobian ellipsoid, if a binary is to be formed without the necessity of an immediate contraction.

There is a question, however, of whether or not such stars, assuming now that they become ellipsoidal upon contraction, could take on the extreme shapes possible in the case of a fictitious homogeneous, liquid, incompressible star. There are several factors which affect the stability of the former stars, but not the latter. They are (1) the expansive tendency of a gas, (2) radiation pressure (of especial

importance in this model), (3) the dynamic tendency to instability because the density does not always decrease toward the center, and, in the opinion of Russell, still more important is (4) the great change in the rate of generation of energy for small changes in temperature. All these are factors which might cause instability to set in long before it would do so in the case of the fictitious body mentioned, and consequently the writer doubts that the Rosseland model could assume the figures of the extremely elongated ellipsoids. It should be clear, however, that nothing definite can be said with regard to the possibility of fission of a star based on this model until its course of evolution, as it contracts, is determined.

SUMMARY

Our discussion has been confined mainly to W Crucis and Capella, two binaries supposed to have been formed by fission, but the results are applicable, obviously, to many other such binaries.

The assumption that these two binaries were formed by fission implies that at some time shortly before this occurred their densities did not exceed 10^{-6} gm/cm^3 anywhere within their interiors, and this result is independent of the model that the parent-stars followed. Because of this it seems very unlikely that their cores could have been liquid. A gaseous star will break up by shedding matter at the Equator, but cannot form a binary by fission if the ratio of the central density to the mean exceeds 2.7. This condition is satisfied in most of the standard models, and therefore they are not subject to fission.

In order that fission may be possible it appears that the following conditions must be satisfied: (*a*) the star is built on the model of Rosseland or something similar, and (*b*) it assumes the forms of the extremely elongated ellipsoids of Jacobi, upon contraction.

There are almost no observational data concerning the law of internal density of stars, with the exception of the eclipsing binary Y Cygni. From a study of the motion of its line of apsides, complicated by the possible presence of a third body, Russell and R. S. Dugan[15] deduce that for the components the ratio of the central to

[15] *Monthly Notices of the Royal Astronomical Society*, **91**, 212, 1930.

the mean density is about 16. Such stars are not susceptible to fission. Y Cygni itself is supposed to have been formed by fission, but it is not necessarily true, of course, that the parent-star was built on the same model as its components.

We have indicated in this paper some dynamical difficulties encountered in assuming that certain classes of binary stars have been formed by fission. The process of fission—if indeed it can occur—must be far more complicated than is usually imagined.

I wish to thank Professor Henry Norris Russell, who suggested the writing of this paper, for the valuable advice and the many criticisms offered during its preparation.

University of Chicago
March 20, 1933

THE DISTRIBUTION OF ABSOLUTE MAGNITUDES AMONG STARS OF SPECTRAL TYPES A, F, G, AND K AS DETERMINED FROM TRIGONOMETRIC PARALLAXES[1]

By GUSTAF STRÖMBERG

ABSTRACT

The distribution of absolute magnitudes of stars of spectral types A, F, G, and K has been determined from the trigonometric parallaxes. Only stars brighter than apparent magnitude 5.5 have been used. For each spectral class the stars were divided into two groups, brighter and fainter than magnitude 4.5. The method consists in solving the integral equation involving the distribution of parallaxes and accidental errors both multiplied by the factor $10^{0.2m}$.

The results are in good agreement with those found by a similar method from the distributions of parallactic and peculiar motions and radial velocities and published in *Mount Wilson Contribution* No. 442.

1. In *Mount Wilson Contribution* No. 442[2] the distribution of absolute magnitudes of stars brighter than apparent magnitude 6.0 and of spectral type B0–M has been published. These determinations were based on reduced parallactic and peculiar motions and on radial velocities. It seemed desirable to the writer to obtain some independent check on the results given. Accordingly, an attempt was made to use a similar method for obtaining the distribution of absolute magnitudes from measured trigonometric parallaxes. In the meantime a paper by R. E. Wilson[3] has appeared, giving the distribution of absolute magnitudes from all obtainable trigonometric, spectroscopic, and dynamic parallaxes. Wilson's luminosity-curves certainly give a good representation for the dwarf stars and the fainter giants, but the result for the more luminous stars, in particular when based directly on trigonometric parallaxes, is questionable.

2. For a long time it has been the custom, when deriving absolute magnitudes from trigonometric parallaxes, simply to drop the negative parallaxes and determine the absolute magnitudes from the positive parallaxes without regard to the errors in the measurements. When the percentage errors are large, the uncertainty in the absolute magnitudes is great, and large systematic errors are introduced. The

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 473.

[2] *Astrophysical Journal*, **75**, 115, 1932.

[3] *Astronomical Journal*, **41**, 169, 1932.

systematic effect does not suddenly set in when the trigonometric parallaxes become zero or negative, but is noticeable also for small positive parallaxes and disappears gradually as the percentage errors become small. No effort has hitherto been made to correct for this effect, and the present study is an attempt to derive the distribution-curves of absolute magnitudes after proper care has been taken to correct for the effect of the accidental errors in the parallaxes.

3. The present analysis is based on trigonometric parallaxes alone and is limited to stars brighter than apparent magnitude 5.5 on the Harvard scale. The study is limited to spectral types A, F, G, and K, since the parallaxes of the B stars are too small and those of the M stars too few to give distribution-curves for the stars of high luminosity. Variable stars have in general been omitted.

In the present study we are not so much concerned with the systematic errors of the different parallax observers as with the systematic errors common to all observers. Much work along this line has been done under the direction of Professor F. Schlesinger, who has been kind enough to give me a complete list of mean parallaxes with individual probable errors, all reduced to a homogeneous system. I take this opportunity to express my great appreciation for this assistance. After this work was begun, Mr. van Maanen[4] made a new and more extended study of the systematic corrections to different series of parallaxes, but there is no appreciable difference between the mean systems, based as they are upon the same data.

4. In the following equations we shall denote the trigonometric parallax of a star by π, its true error by ϵ, the true absolute magnitude by M, and the apparent magnitude by m.

We have, then, the equation

$$M=m+5+5\log(\pi+\epsilon)\,,$$

or

$$(\pi+\epsilon)10^{0.2m}=0.1\cdot 10^{0.2M}\,. \tag{1}$$

Denote

$$\left.\begin{aligned}\pi\ 10^{0.2m}&\ \text{by}\ x\\ \epsilon\ 10^{0.2m}&\ \text{by}\ y\\ 0.1\cdot 10^{0.2M}&\ \text{by}\ z\end{aligned}\right\} \tag{2}$$

[4] *Mt. Wilson Contr.*, No. 474; *Astrophysical Journal*, **78**, 189, 1933.

and we have

$$z = x + y \ . \tag{3}$$

If now we study the existing correlations, we find that x and y are strongly correlated through their direct common dependence on m; x and z are dependent on each other through correlations between m and M and between π and M; while y and z are connected only through the correlation between m and M. If we limit the range in m, we may regard y and z as independent variables, determining the dependent variable x through the relation

$$x = z - y \ . \tag{4}$$

We have now to deal with three distribution functions $F_1(x)dx$, $F_2(y)dy$, and $F_3(z)dz$, of which $F_1(x)dx$ is known numerically from the observed data, $F_2(y)dy$ can be computed from an assumed distribution of the errors ϵ, and $F_3(z)dz$ is the unknown distribution that will give us the distribution of M.

The three distribution functions are connected by the integral equation

$$\frac{F_1(x)dx}{N_1} = \frac{dx}{N_2 N_3} \int_0^{\infty} F_2(z-x) F_3(z) dz \ . \tag{5}$$

N_1, N_2, and N_3 are the total numbers of stars in the distributions F_1, F_2, and F_3, respectively, numbers which in the present case are all alike and equal to N.

To solve the integral equation we shall use the same method as was applied in *Mount Wilson Contribution* No. 395[5] and start from an assumed distribution of z, denoted by $f_3(z)dz$. From this distribution and that of y we derive numerically an expected distribution of x, denoted by $f_1(x)dx$, from the equation

$$f_1(x)dx = \frac{dx}{N} \int_0^{\infty} F_2(z-x) f_3(z) dz \ . \tag{6}$$

From the differences between observed and computed values of the distribution of x for different intervals in x we can derive corrections $\Delta f_3(z)dz$ to the assumed distribution $f_3(z)dz$ for a particular small

[5] *Astrophysical Journal*, **71**, 163, 1930.

range in z. The equation of condition for a range in x from x_1 to x_2, the mean of which is denoted by x_{12}, is

$$\Delta f_3(z_{12})\sum_{z_1-x_{12}}^{z_2-x_{12}} F_2(y)dy+\Delta f_3(z_{23})\sum_{z_2-x_{12}}^{z_3-x_{12}} F_2(y)dy+\text{etc.}=\frac{N}{x_2-x_1}\sum_{x_1}^{x_2}\Delta f_1(x)dx\ . \quad (7)$$

A similar equation is obtained for the next interval x_2–x_3, and so on until the whole range in x has been utilized.

In the computation of $F_2(y)dy$ the individual values of the probable errors as given by Schlesinger were multiplied by $10^{0.2m}$. Within finite intervals of these reduced probable errors, the distribution of the actual errors was derived on the assumption that the errors in the parallaxes are distributed according to a normal error-curve, and the partial distributions were then summed for different values of y.

The intervals in x used for the equations of condition cannot be made uniform throughout. It was found convenient to take uniform intervals equal to 0.01 from the negative limit up to $x=+0.05$, which corresponds to $M=-1.5$. From this point on, intervals uniform in terms of M and equal to 0.5 were used.

On account of the non-uniformity of the intervals in x it was not possible to determine all the corrections $\Delta f_3(z)$ simultaneously, as in *Mount Wilson Contribution* No. 395 and subsequent investigations. Four different solutions had to be made, and in each the difference between the values of M for which the corrections $\Delta f_3(z)$ were to be determined was 2 mag. In the subsequent solution a shift of half a magnitude was made, resulting in a new set of coefficients in the equations of condition. The individual equations of solution were given weights inversely proportional to $f_1(x)dx/\Delta x$.

After the distribution $F_3(z)dz$ had been found, it was converted into a distribution of M with a uniform interval $\Delta M=0.5$. A new expected distribution of x was then computed and compared with the observed distribution. In nearly all cases the second approximation for $F(M)dM$ was entirely satisfactory and did not warrant any further attempt at improvement.

For the more luminous stars it was found necessary, in order to stabilize the least-squares solution, to assume arbitrarily that there are no stars brighter than $M=-5.5$. The condition was also introduced that the frequency-curve cannot be negative.

5. The results of this analysis are given in Tables I–IV and in Figures 1 and 2. The stars were divided into four spectral divisions: A, F, G, and K. If a star was found to have been classified as of

TABLE I

OBSERVED AND COMPUTED DISTRIBUTIONS OF $x=\pi 10^{0.2m}$ FOR STARS BRIGHTER THAN $m=4.5$

M	x	A		F		G		K	
		O	C	O	C	O	C	O	C
	−0.20								
	−0.19	0	1	......	......	......	......	......	......
	−0.18	0	0	......	......	......	......	......	......
	−0.17	0	0	......	......	......	......	......	......
	−0.16	0	0	......	......	......	......	......	......
	−0.15	0	1	......	......	......	......	......	......
	−0.14	7	0	......	......	......	1	6	1
	−0.13	0	1	0	1	0	1	0	1
	−0.12	0	1	0	1	0	1	0	1
	−0.11	0	1	0	1	0	1	0	1
	−0.10	0	1	0	1	0	1	0	1
	−0.09	0	2	0	1	0	2	0	2
	−0.08	0	2	0	2	0	2	0	2
	−0.07	0	3	0	2	18	3	6	2
	−0.06	0	3	0	3	0	4	6	2
	−0.05	7	4	0	4	0	5	0	3
	−0.04	0	4	0	5	0	7	6	4
	−0.03	22	5	0	6	4	9	0	5
	−0.02	0	7	0	7	18	10	0	7
	−0.01	15	8	13	9	31	13	17	10
	0.00	4	11	13	10	31	15	20	10
	+0.01	11	14	25	12	22	18	14	12
	+0.02	22	17	0	14	9	21	30	13
	+0.03	8	19	38	15	27	23	20	18
−2.0...	+0.04	34	24	25	20	27	27	23	24
−1.5...	+0.05	22	28	38	26	22	30	31	32
−1.0...	+0.063	49	36	0	29	40	37	51	45
−0.5...	+0.079	48	50	38	36	27	52	49	63
0.0...	+0.100	82	75	51	47	53	82	89	99
+0.5...	+0.126	71	98	63	57	187	106	169	133
+1.0...	+0.158	131	117	63	60	85	112	123	142
+1.5...	+0.200	116	130	38	62	94	108	146	137
+2.0...	+0.251	134	137	101	70	89	82	74	96
+2.5...	+0.316	142	125	101	92	45	53	46	66
+3.0...	+0.398	75	64	101	112	27	32	34	25
+3.5...	+0.501	0	10	152	123	36	28	11	13
+4.0...	+0.631	0	1	76	89	27	28	6	6
+4.5...	+0.794	......	......	13	55	27	29	0	1
+5.0...	+1.000	......	......	51	25	27	24	0	0
+5.5...	+1.26	......	......	0	3	9	16	6	4
+6.0...	+1.58	......	......	......	......	0	10	6	8
+6.5...	+2.00	......	......	......	......	18	5	11	11
+7.0...	+2.51	......	......	......	......	0	2	......	......

TABLE II

OBSERVED AND COMPUTED DISTRIBUTIONS OF $x=\pi 10^{0.2m}$ FOR STARS BETWEEN $m=4.5$ AND 5.5

M	*x*	A		F		G		K	
		O	C	O	C	O	C	O	C
	−0.30	......	......	......	......	5	......	0	1
	−0.29	......	......	......	......	6	1	6	0
	−0.28	......	......	......	......	0	0	0	0
	−0.27	......	......	......	......	0	1	0	1
	−0.26	......	......	......	......	0	1	0	0
	−0.25	0	1	10	1	0	1	0	1
	−0.24	0	0	10	0	0	1	0	1
	−0.23	0	0	0	0	0	1	0	1
	−0.22	0	1	0	1	0	1	0	1
	−0.21	0	1	0	0	0	2	0	1
	−0.20	0	1	0	1	5	2	0	1
	−0.19	0	1	0	0	6	2	13	2
	−0.18	0	1	0	1	5	2	7	2
	−0.17	14	1	0	0	6	3	0	2
	−0.16	5	2	0	1	0	3	0	3
	−0.15	0	2	0	1	0	4	0	3
	−0.14	0	2	0	1	0	5	0	3
	−0.13	0	2	0	1	0	5	10	4
	−0.12	0	3	0	1	0	6	10	5
	−0.11	0	3	0	2	5	6	16	5
	−0.10	0	4	0	2	22	7	10	6
	−0.09	9	5	10	2	16	8	26	7
	−0.08	0	5	20	2	5	9	13	8
	−0.07	0	6	0	3	5	11	20	9
	−0.06	0	7	0	3	0	11	6	10
	−0.05	9	8	10	3	11	12	0	12
	−0.04	0	9	0	3	22	13	7	13
	−0.03	5	10	0	3	5	14	0	14
	−0.02	18	11	0	3	5	15	0	16
	−0.01	5	12	0	3	5	15	0	18
	0.00	37	13	0	3	16	16	33	20
	+0.01	9	15	0	3	33	17	7	22
	+0.02	19	17	0	3	44	17	26	23
	+0.03	42	20	0	3	22	17	30	26
−2.0...	+0.04	14	24	10	4	5	19	40	27
−1.5...	+0.05	28	29	0	7	27	22	36	33
−1.0...	+0.063	28	40	0	9	22	27	26	41
−0.5...	+0.079	79	61	30	17	33	35	60	60
0.0...	+0.100	125	83	10	24	38	38	106	78
+0.5...	+0.126	93	104	40	35	27	43	66	93
+1.0...	+0.158	78	123	50	46	49	50	76	103
+1.5...	+0.200	106	136	100	72	82	56	116	98
+2.0...	+0.251	148	118	75	130	54	62	66	80
+2.5...	+0.316	88	74	135	163	65	65	73	54
+3.0...	+0.398	23	34	240	178	76	65	40	30
+3.5...	+0.501	18	9	140	154	44	65	7	13
+4.0...	+0.631	0	2	75	80	60	63	0	3
+4.5...	+0.794								

TABLE II—*Continued*

M	x	A		F		G		K	
		O	C	O	C	O	C	O	C
+4.5...	+0.794	....	....	35	28	82	58	0	0
+5.0...	+1.00	....	....	0	3	33	48	7	2
+5.5...	+1.26	....	....	....	....	43	36	7	7
+6.0...	+1.58	....	....	....	....	11	18	7	12
+6.5...	+2.00	....	....	....	....	0	1	13	12
+7.0...	+2.51	....	....	....	....	....	....	7	9
+7.5...	+3.16	....	....	....	....	....	....	7	4
+8.0...	+3.98								

different types at Harvard and Mount Wilson, it was included in both groups, and there is thus a slight overlapping between the spectral groups. Within each spectral class the stars were divided into two groups: those of apparent magnitude brighter than 4.50 and those between magnitudes 4.51 and 5.50.

TABLE III

DISTRIBUTION OF $y=\epsilon\, 10^{0.2m}$

y	$m \leqq 4.50$				$m=4.51-5.50$			
	A	F	G	K	A	F	G	K
0.00...........	160	142	152	151	82	82	80	78
0.02...........	108	114	116	116	77	76	76	75
0.04...........	77	86	86	85	68	68	69	68
0.06...........	52	58	55	56	58	60	60	60
0.08...........	35	38	36	36	49	49	50	50
0.10...........	24	24	23	23	40	40	41	42
0.12...........	17	16	14	13	32	31	32	33
0.14...........	11	9	10	9	24	24	25	24
0.16...........	6	6	6	5	19	18	18	20
0.18...........	4	3	2	3	15	14	15	13
0.20...........	2	2	....	2	10	11	11	10
0.22...........	2	1	....	1	7	9	7	8
0.24...........	1	1	....	....	6	6	6	6
0.26...........	1	....	....	....	5	5	5	4
0.28...........	....	....	....	....	3	3	3	3
0.30...........	....	....	....	....	2	3	1	2
0.32...........	....	....	....	....	2	1	1	2
0.34...........	....	....	....	....	1	....	....	1
0.36...........	....	....	....	....	....	....	....	1
0.38...........								

Table I gives the observed and computed distributions of $x=\pi 10^{0.2m}$ for stars brighter than $m=4.5$, while Table II gives the same distributions for stars between magnitudes 4.5 and 5.5. As an illustration, the results for spectral type K are plotted in Figure 1, which

TABLE IV

DISTRIBUTION OF ABSOLUTE MAGNITUDE M

	A			F			G			K		
M \ m	<4.5	>4.5	<5.5	<4.5	>4.5	<5.5	<4.5	>4.5	<5.5	<4.5	>4.5	<5.5
−5.5	.....	.....	.....	.....	3	2	.....	.....	.....	.....	1	.....
−5.0	.....	.....	.....	.....	14	8	15	25	19	5	11	7
−4.5	.....	.....	.....	.....	17	9	33	54	42	15	27	21
−4.0	.....	.....	.....	.....	17	10	40	77	57	35	33	34
−3.5	.....	.....	.....	8	8	7	37	83	57	20	31	25
−3.0	7	.....	4	27	0	12	12	78	42	0	15	7
−2.5	25	.....	14	36	0	16	0	32	15	0	2	1
−2.0	47	4	28	40	0	18	0	0	0	0	0	0
−1.5	70	23	49	47	0	21	6	0	3	31	35	33
−1.0	79	70	75	56	0	24	70	5	41	100	110	105
−0.5	86	115	99	75	0	33	135	18	82	143	148	146
0.0	102	161	129	69	0	30	175	30	110	181	150	166
+0.5	119	186	149	50	16	31	120	37	83	180	123	154
+1.0	140	176	156	41	52	47	91	43	69	130	90	112
+1.5	165	139	153	59	93	78	72	50	63	75	65	70
+2.0	120	85	104	90	122	108	29	58	42	30	47	38
+2.5	40	35	38	125	198	166	22	62	40	15	35	24
+3.0	.....	5	2	115	231	180	28	62	43	11	22	16
+3.5	.....	1	.....	81	146	118	31	63	45	6	5	5
+4.0	.....	.....	.....	57	65	61	29	63	44	0	0	0
+4.5	.....	.....	.....	22	16	19	23	60	40	0	0	0
+5.0	.....	.....	.....	2	2	2	15	48	30	4	3	4
+5.5	.....	.....	.....	.....	.....	.....	10	35	21	8	8	8
+6.0	.....	.....	.....	.....	.....	.....	5	17	11	11	12	11
+6.5	.....	.....	.....	.....	.....	.....	2	.....	1	.....	12	6
+7.0	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	10	5
+7.5	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	5	2
+8.0												
No.	134	108	242	79	100	179	112	92	204	175	151	326

shows that the observed data are well represented by the computed curve.

Table III shows the distribution of $y=\epsilon 10^{0.2m}$ for the different groups of stars, where ϵ is the actual error of a trigonometric parallax.

Table IV gives the computed distributions of the absolute magni-

tudes M for the different groups studied. The total numbers of stars used are indicated at the bottom. The combined distributions for $m \leqslant 5.5$ are also derived for the four spectral divisions.

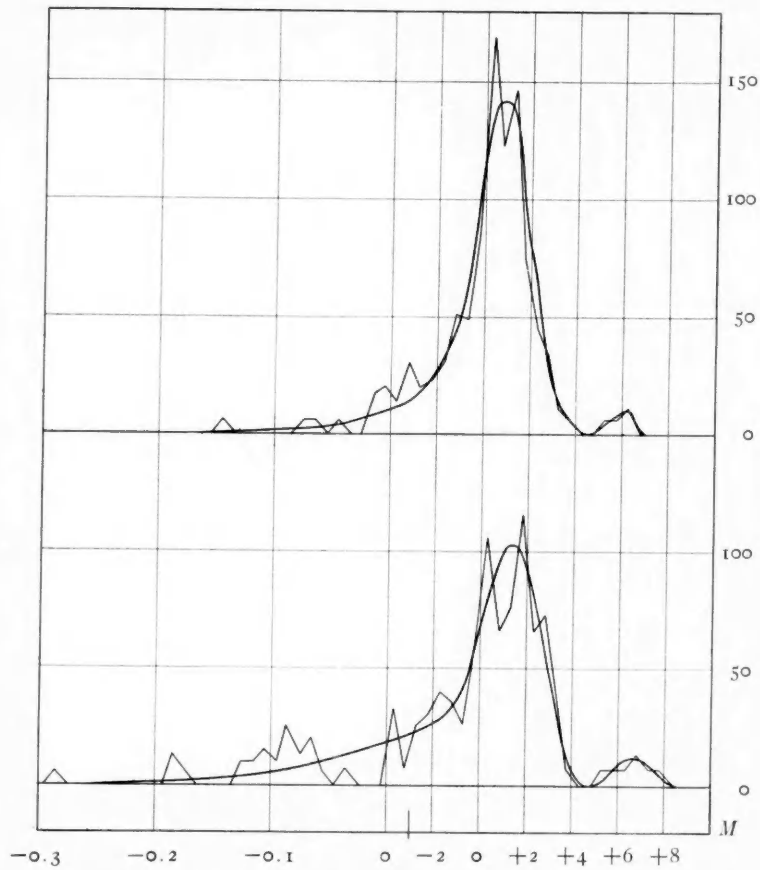

FIG. 1.—Distribution of $x = \pi\ 10^{0.2m}$ for stars of spectral type K and of apparent magnitude brighter than 4.5 (upper curve), and between 4.5 and 5.5 (lower figure). The broken lines indicate observed values, whereas the continuous curves are computed from the distribution of absolute magnitudes and accidental errors.

In all the distributions given in the tables the numbers of stars have been reduced to a total of 1000.

6. The final distributions of M for $m \leqslant 5.5$ are shown graphically in Figure 2. Except for the stars of highest luminosities, the frequencies are quite similar to those found by Wilson from the aggre-

gate of all known absolute magnitudes. For comparison with the results from proper motions and radial velocities given in *Mount*

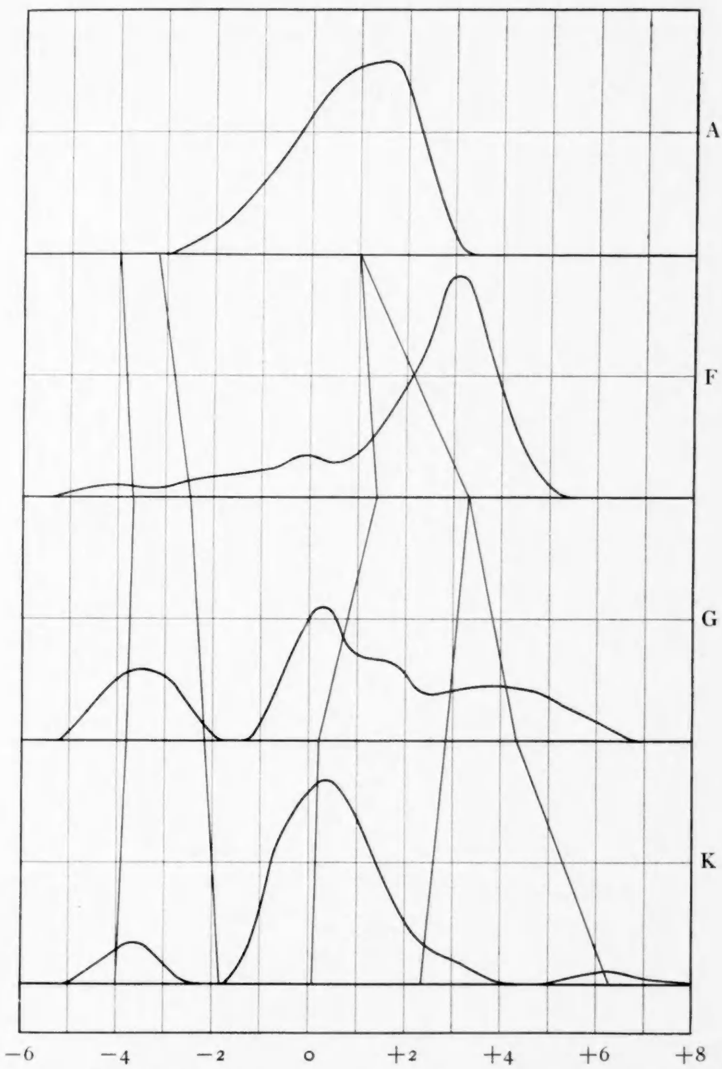

FIG. 2.—Distribution of absolute magnitudes for stars of spectral types A, F, G, and K, and of apparent magnitude brighter than 5.5.

Wilson Contribution No. 442 I have in the present diagram copied the lines which were found to represent the maximum frequencies in

that investigation. The five sequences previously found were denoted by the terms "supergiants," "bright giants," "normal giants," "faint giants," and "dwarfs" (main-sequence stars). As a whole, the agreement with the previous results is satisfactory, provided we remember that the accuracy from trigonometric parallaxes for the highly luminous stars is inferior to and for the intrinsically fainter stars superior to that of results from proper motions and radial velocities. The main differences between the present and the previous results are that the bulk of the F stars does not show any decided division into two groups, and that the frequency for the F stars does not quite vanish for absolute magnitudes around -1. We also do not find any supergiants among the A stars, although we know that such stars certainly exist. A very small systematic negative correction to the parallaxes or a slight decrease in the assumed probable errors would be sufficient to provide the small number of supergiants actually observed.

In conclusion we may say that the absence of K stars of absolute magnitude around $+4.5$ is now well established. The existence of "faint giants" (formerly called "subgiants") of spectral classes K and G is also rather well established. The zero frequency for G stars having values of M around -1.5 is very probable, provided variable stars are omitted. The existence of "supergiants" and "bright giants" as two separate groups is still not proved.

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SYSTEMATIC ERRORS IN TRIGONOMETRIC PARALLAXES AS A FUNCTION OF RIGHT ASCENSION[1]

By ADRIAAN VAN MAANEN

ABSTRACT

From a comparison of the trigonometric parallaxes derived at the Mount Wilson, Allegheny, McCormick, Yerkes, Sproul, Greenwich, Johannesburg, and Cape of Good Hope Observatories with the recently determined system of Mount Wilson spectroscopic parallaxes, it was found that all eight series are affected by systematic errors which are a function of right ascension. The corrections derived are given in Table VII. The application of these corrections produces a marked improvement in the agreement between the different series *inter se*.

Five years ago[2] a system of systematic errors for seven series of modern trigonometric parallaxes was derived which showed these errors to be in each case a function of right ascension. For several reasons it is now desirable to repeat the determination. To begin with, a more satisfactory set of reductions from relative to absolute parallax can now be used; further, the material of all observers has increased considerably, and two new extensive series of trigonometric parallaxes have become available from observatories in the Southern Hemisphere, viz., Johannesburg and Cape of Good Hope; finally, a much more consistent system of spectroscopic parallaxes can now be used, which, while not yet published, was kindly put at my disposal by Mr. Adams.

In order to see if a discussion of systematic errors is warranted, I first made a direct comparison between the two most extensive series of trigonometric parallaxes available, those of Allegheny and of McCormick. At present 442 stars have been measured at both observatories. The mean differences for each hour in right ascension are given in Table I. (The unit used in all the tables is 0ʺ001.) The results are also shown by the broken line in Figure 1. In order to smooth the results, the mean of three successive hours has been used here, as has been done all through this paper in the plots and in the computation of the constants. There can be no doubt, I think, that

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 474.

[2] *Mt. Wilson Contr.*, No. 357, 1928.

these differences are a function of right ascension. With each three-hourly mean used as the second member, M, of an equation of the form

$$a_0+a_1 \cos \varphi+b_1 \cos 2\varphi+c_1 \cos 4\varphi+a_2 \sin \varphi+b_2 \sin 2\varphi +c_2 \sin 4\varphi=M \ , \quad (1)$$

the constants were determined by a least-squares solution. The inclusion of the two terms containing 4φ, which were not used in

TABLE I

HOURLY DIFFERENCES, π ALLEGHENY $-$ π McCORMICK

a	Alleg.—McC.	n	a	Alleg.—McC.	n
0^h	+2.4	13	12^h	−5.5	6
1	−8.3	18	13	−9.0	9
2	−2.3	18	14	−0.3	15
3	+5.1	13	15	−3.1	21
4	−6.3	12	16	+1.0	25
5	+7.3	16	17	−4.7	28
6	+6.2	18	18	−2.6	25
7	+7.4	18	19	−3.3	45
8	−1.6	14	20	−8.3	33
9	+3.4	14	21	−8.8	22
10	−0.1	11	22	+0.5	25
11	−3.1	8	23	−4.1	15

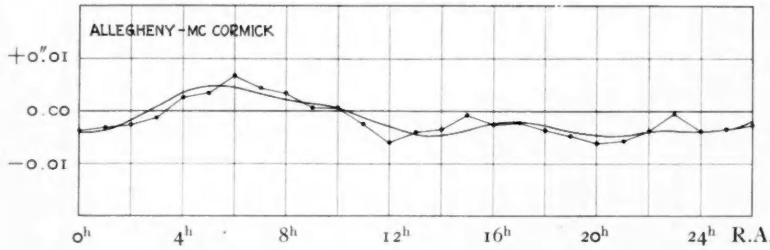


FIG. 1.—Mean differences in parallax grouped according to right ascension

my former paper, will be discussed farther on. The values of the constants are:

$$a_0=-1.4 \qquad a_1=-0.5 \qquad a_2=+3.6$$
$$b_1=-2.2 \qquad b_2=+0.1$$
$$c_1=+0.1 \qquad c_2=-0.9$$

By substituting the constants a_0, a_1 . . . . c_2 into the equations of condition, a smooth curve can be drawn which represents the mean differences extremely well. On applying the corrections called for by these constants, the $\Sigma\delta^2$ of the three-hourly differences drops from 328 to 55, a reduction so large as to remove any doubt as to the usefulness of the formula. The differences read from the smooth curve run from $-0''.0046$ to $+0''.0047$, quantities which should not be neglected if there is a way of removing them, especially as they seem clearly to be systematic.

As before, there is no set of true parallaxes available which may be used as a reference system for the correction of the observed trigonometric parallaxes, and the best system for comparison seems still to be the spectroscopic parallaxes, now considerably improved, as these are not likely to show any systematic effect with right ascension. The present system of spectroscopic parallaxes is based largely on proper motions. The B- and early A-type stars, whose spectroscopic parallaxes formerly were based on the assumption of a constant absolute magnitude for any subdivision, have been omitted; all the spectroscopic parallaxes used are based, therefore, on relative line intensities.

But, first of all, we have to reduce the relative trigonometric parallaxes to absolute values. For the faint stars used at Mount Wilson for comparison purposes, good values of the mean parallax were derived during last year by H. C. Willis. Willis' results for different galactic latitudes give for the ratio, π_{90} to π_0, the value 2.15, an amount considerably smaller than that found by van de Kamp,[3] which was used in my former discussion. As there was some evidence "that the correction used for the reduction from relative to absolute parallaxes may be too large for high galactic latitudes,"[4] and as Willis' value agrees well with the results given for the stars of magnitude 9, 10, and 11, derived by van Rhijn and Bok,[5] this ratio has been used to derive the system of corrections used in the present paper. These corrections are given in Table II for different magnitudes and galactic latitudes.

[3] *Astronomical Journal*, **37**, 191, 1927.

[4] *Mt. Wilson Contr.*, No. 357, p. 15, 1928.

[5] *Publications of the Kapteyn Astronomical Laboratory at Groningen*, No. 45, 1931.

The ten series of modern trigonometric parallaxes thus corrected were compared with the new list of about four thousand spectroscopic parallaxes by Adams. The data used are summarized in Table III, which gives the type of instrument and the number of stars observed for which a spectroscopic parallax is available (next to last column). In all, 2672 differences $\pi_t - \pi_s$ are now available; the

TABLE II

REDUCTION FROM RELATIVE TO ABSOLUTE PARALLAX

(Unit, 0."001)

Mag. 10		Mag. 11		Mag. 12		Mag. 13	
0°–35°	3	0°–31°	2	0°–45°	2	0°– 5°	1
36 –54	4	32 –57	3	46 –79	3	6 –56	2
55 –75	5	58 –90	4	80 –90	4	57 –90	3
76 –90	6						

TABLE III

COMPILATION OF MATERIAL AVAILABLE

Mount Wilson 60-inch	Photographic	Reflector	189	184
Mount Wilson 100-inch	Photographic	Reflector	16	15
Allegheny	Photographic	Refractor	750	745
McCormick	Photovisual	Refractor	737	721
Yerkes	Photovisual	Refractor	225	214
Sproul	Photovisual	Refractor	200	192
Greenwich	Photographic	Refractor	162	162
Johannesburg	Photographic	Refractor	263	250
Cape Photographic	Photographic	Refractor	96	92
Van Vleck	Photovisual	Refractor	34	29

mean systematic difference is +0."0014. Since the values of the spectroscopic parallaxes are rather uncertain when large (a good deal of this uncertainty is probably due to the unreliability of the magnitudes of the fainter stars of large proper motion), the material was subdivided into groups for which the spectroscopic parallaxes were >0."400, 0."200–0."399, 0."100–0."199, 0."050–0."099, and <0."050. The first two groups show a considerable dispersion in the differences $\pi_t - \pi_s$ and have therefore been excluded from the discussion. The last column of Table III gives the total number of stars available for each observer in the restricted list used in the present paper. The total for this list is 2604 differences, $\pi_t - \pi_s$, with a mean

value of +0″.0009. This mean systematic difference is too small to have an appreciable influence on the results.

It was first thought that there might possibly be a systematic difference for different spectral types. All observers together give for the different types the results tabulated. While there is some

Type	Difference	Number
Late A	−0″.0021	162
F	+ .0005	661
G	− .0002	769
K	+ .0027	697
M	+0.0015	315
Total	....	2604

TABLE IV

MEAN DIFFERENCES $\pi_t - \pi_s$ FOR DIFFERENT SPECTRAL TYPES

Sp.	MW 60		Alleg.		McC.		Yerkes	
A	−8.0	4	−5.4	53	−7.9	44	−3.1	17
F	+0.3	44	−2.4	217	−2.2	168	+3.3	45
G	+2.0	47	−1.4	223	0.0	213	−0.5	61
K	+6.5	51	+1.0	196	+1.8	212	+0.3	48
M	+7.0	38	−0.6	56	+3.1	84	+2.4	43

Sp.	Sproul		Greenw.		Johan.		Cape Ph.	
A	+10.6	21	+1.5	6	+ 7.1	15	+21.0	1
F	+ 6.3	73	+0.9	38	7.1	49	+ 8.4	23
G	− 0.1	57	+0.1	53	3.2	71	− 0.8	37
K	+ 4.2	31	−1.8	46	12.2	76	+ 3.2	21
M	− 5.2	10	−0.8	19	+ 3.1	39	+ 9.8	10

indication that in general the spectroscopic parallaxes for the early-type stars may be too large and those for the late types too small, the differences for different observers are so varied that a part of the run is probably due to the trigonometric parallaxes. The results for the eight longer series are given in Table IV. As the number of A stars is relatively small, it was decided to keep the material included in the list.

By plotting the curves $\pi_t - \pi_s$ for different observatories (Fig. 2), it is clearly seen that all of them exhibit differences which seem to be

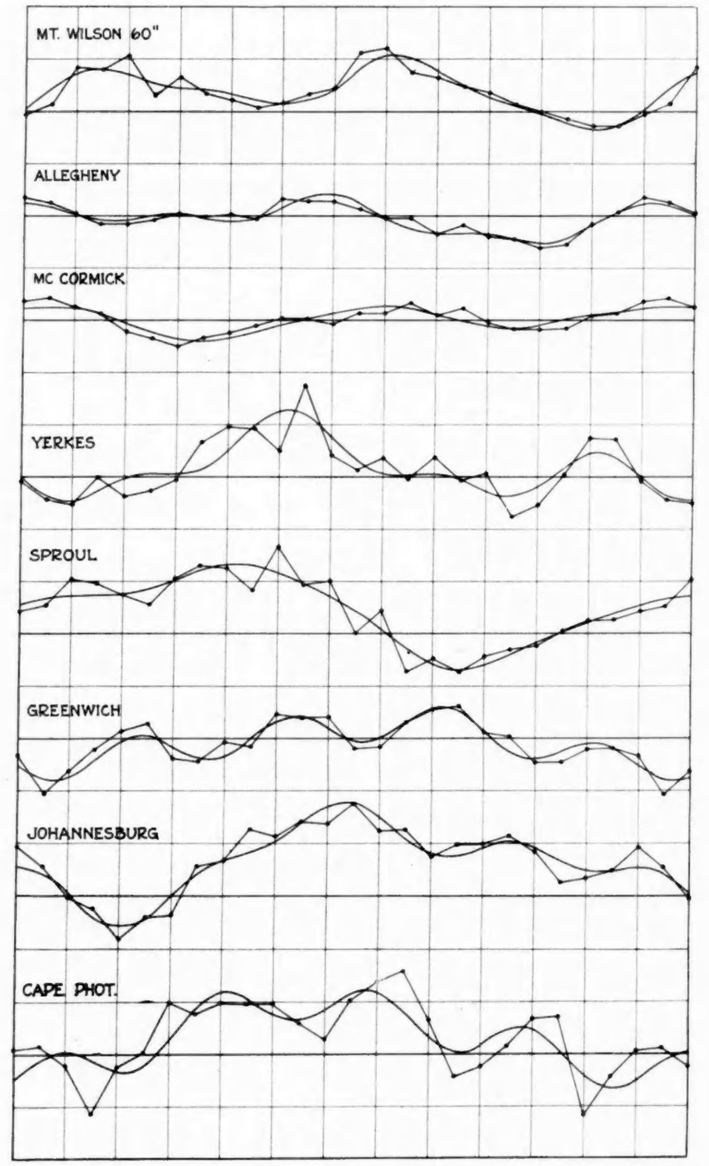


FIG. 2.—Mean differences $\pi_t - \pi_s$ grouped according to right ascension. Scale of ordinates: 1 square = 0".01.

a function of right ascension. In most cases there are two decided maxima and minima; in a few there are one strong and one weak maximum and minimum, while in three cases there is definite evidence of four maxima and minima. Since in general these maxima and minima for the different observers do not fall in the same hours, it would be illogical to blame the spectroscopic parallaxes for the phenomenon. It seems sounder to think of possible errors in the trigonometric parallaxes. This point is emphasized by the fact that the series showing the strongest effect of four maxima and minima is from an observatory where no guiding is done during the exposures at the telescope. If then, as was suggested in a former paper,[6] part of the systematic errors may be due to a difference in the rate of the clock for morning and evening exposures, we might expect, roughly, six-hour intervals in the values of $\pi_t - \pi_s$. Even when the plates are taken with guiding, some influence due to this cause may be found, and it was accordingly decided to include the terms $c_1 \cos 4\varphi$ and $c_2 \sin 4\varphi$ in the derivation of the constants.

Table V gives the hourly mean differences for the eight longer series. For the Mount Wilson 100-inch and for Van Vleck, only 15 and 29 stars, respectively, are available, numbers too small for deriving systematic differences as a function of right ascension.

The values of Table V have been used to form the overlapping means for three successive hours which are plotted in Figure 2 as broken curves. The values of the constants for formula (1) are given in Table VI, and the smooth curves based on them are reproduced in Figure 2.

The closeness with which the curves generally represent the observations is striking, and there seems to be little doubt that we should accept the seasonal differences as real. The corrections to be applied to each series of measures as found from the formula are given for each hour in Table VII. At present the corrections for the Mount Wilson 100-inch and for Van Vleck may be taken as $+0\overset{''}{.}006$ and $+0\overset{''}{.}007$, respectively. How much improvement the application of the corrections gives may be seen from Table VIII, which shows the values of $\Sigma\delta^2$, derived in the way described for Allegheny–McCormick on page 191.

[6] *Op. cit.*, p. 4.

TABLE V

Comparison of Trigonometric and Spectroscopic Parallaxes

α	MW 60 − Spec.		Alleg. − Spec.		McC. − Spec.		Yerkes − Spec.		Sproul − Spec.		Greenw. − Spec.		Johan. − Spec.		Cape Ph. − Spec.	
0^h	− 1.0	7	+5.3	27	+4.1	25	− 8.0	4	+ 1.2	6	−10.2	6	+ 8.1	11	+ 5.9	7
1	+ 0.4	7	+3.2	27	+5.2	29	−12.6	8	+ 5.2	4	−11.6	5	+ 6.2	6	+ 5.5	4
2	+ 3.0	12	−0.7	27	+3.3	34	+ 6.2	8	+ 7.4	12	−10.2	6	+ 0.3	10	−12.0	4
3	+28.7	6	−1.8	22	−1.2	27	− 8.6	9	+16.7	9	+ 0.2	8	− 8.6	7	+ 3.0	1
4	− 1.8	6	−2.2	23	+1.5	29	+ 4.0	7	+ 6.0	9	+ 2.8	6	− 1.4	7	−16.7	3
5	+ 5.5	6	−0.4	29	−6.2	31	− 4.5	10	− 3.0	7	+ 2.0	5	−12.5	11	+ 6.0	4
6	+ 5.0	8	−0.5	36	−4.8	29	− 6.0	8	+14.0	7	−13.0	6	+ 7.0	7	+ 6.5	4
7	+ 9.7	7	+2.8	36	−3.8	26	+11.4	7	+21.0	7	+ 0.5	6	− 1.8	12	+14.6	5
8	− 2.7	9	−3.2	25	−1.6	32	+15.5	8	+ 3.9	7	− 1.1	8	+14.4	9	+ 2.6	7
9	+ 1.5	8	+0.2	21	−2.1	25	− 2.0	5	+12.9	7	− 0.6	5	+12.5	6	+22.5	2
10	+ 5.0	6	+1.8	17	+0.8	26	+11.5	2	+ 9.0	5	− 4.0	4	+10.2	6	+14.5	4
11	− 0.7	7	+9.9	13	+2.5	25	+26.0	1	+29.8	5	+11.0	10	+10.5	4	+ 1.2	6
12	+ 8.2	4	−0.9	18	−3.8	17	+19.3	3	− 7.0	6	+ 0.6	11	+19.1	8	+ 3.0	3
13	+10.0	4	+1.5	17	−2.6	15	− 5.6	7	+10.6	9	+ 0.7	9	+10.6	10	+ 4.2	5
14	+13.3	7	+2.5	26	+7.6	25	− 0.2	5	− 7.3	7	−14.5	4	+23.2	10	+18.3	6
15	+11.9	8	−4.2	34	−2.3	27	+ 9.3	15	+ 9.8	5	+ 1.8	9	+ 5.5	13	+22.5	2
16	+ 0.7	11	+1.0	43	+5.1	29	−15.2	10	−23.8	5	+12.4	9	+11.9	12	−14.0	1
17	+ 8.2	13	−7.8	44	0.0	40	+11.0	15	− 1.2	6	+ 3.1	10	+ 5.8	17	+ 3.8	5
18	+ 4.3	6	+1.3	47	+2.1	32	− 3.2	13	− 2.3	10	+ 3.2	8	+13.2	13	−15.0	3
19	− 3.0	9	−5.7	56	−2.4	50	− 6.9	14	− 7.3	13	− 7.8	4	+11.4	18	− 1.8	5
20	+ 3.6	7	−9.4	44	−4.1	48	−13.0	14	+0.7	13	+ 1.3	10	+ 9.5	14	+13.8	5
21	+ 0.8	4	−3.1	40	+2.5	29	+ 4.3	13	+ 0.8	9	−19.3	3	+ 2.8	11	+10.0	2
22	− 5.0	13	−4.0	41	−1.8	39	+ 8.0	18	+ 0.5	11	− 7.2	5	− 4.1	14	−33.0	1
23	− 1.4	9	+2.6	32	+2.3	32	+11.5	10	+ 5.5	13	+13.6	5	+11.5	14	−19.7	3

The proof of the pudding is the eating. It was therefore thought worth while to see how the different series of parallaxes are affected

TABLE VI

Series	a_0	a_1	b_1	c_1	a_2	b_2	c_2
MW 60	+3.9	−2.4	−0.5	−0.6	+0.9	+4.6	+1.1
Alleg.	−0.6	−0.9	+2.4	+1.4	+2.0	+0.4	−0.3
McC.	0.0	+0.4	+2.1	−0.3	−1.2	+1.4	0.0
Yerkes	+1.9	−3.9	+2.4	−0.4	+1.7	−3.4	−2.6
Sproul	+4.4	−0.9	+2.2	−0.1	+8.1	−2.2	+0.6
Greenw.	−0.8	−3.5	−0.8	0.0	−1.7	+0.4	−3.2
Johan.	+6.7	−5.8	+3.3	+1.3	−4.6	−2.1	+2.1
Cape Ph	+3.1	−6.7	−0.2	−1.4	+0.2	−1.1	+3.6

TABLE VII

CORRECTIONS TO BE APPLIED TO THE TRIGONOMETRIC PARALLAXES

α	MW 60	Alleg.	McC.	Yerkes	Sproul	Greenw.	Johan.	Cape Ph.
0^h	− 0.4	−2.3	−2.2	0.0	− 5.6	+5.1	− 5.5	+ 5.2
1	− 4.4	−1.7	−2.4	+ 3.6	− 6.9	+7.9	− 4.2	+ 1.7
2	− 7.3	−0.1	−2.2	+ 4.6	− 7.3	+7.5	− 0.5	− 0.1
3	− 8.0	+0.8	−1.1	+ 2.7	− 7.5	+4.1	+ 4.1	+ 1.3
4	− 7.0	+0.6	+0.5	+ 0.3	− 7.5	+0.6	+ 5.9	+ 3.4
5	− 5.6	−0.2	+2.4	− 0.8	− 8.6	−0.4	+ 4.4	+ 2.6
6	− 4.7	−0.4	+3.6	− 0.8	−10.2	+1.7	− 0.1	− 2.1
7	− 4.2	+0.4	+4.0	− 1.6	−12.2	+3.8	− 4.2	− 8.2
8	− 3.4	+1.0	+3.3	− 5.1	−13.1	+3.2	− 6.9	−11.6
9	− 2.2	+0.4	+2.3	− 9.7	−13.1	−0.1	− 8.3	−10.5
10	− 1.5	−1.7	+0.8	−12.8	−11.7	−3.5	−10.3	− 7.5
11	− 2.4	−3.7	−0.2	−12.0	− 9.9	−4.1	−13.8	− 6.3
12	− 5.2	−4.1	−1.4	− 7.8	− 7.4	−1.9	−17.1	− 8.2
13	− 8.6	−2.5	−2.2	− 3.2	− 4.5	+0.3	−17.8	−11.1
14	−10.7	+0.3	−2.8	− 0.6	− 0.9	−0.1	−15.1	−11.5
15	−10.2	+2.4	−2.3	− 0.5	+ 2.9	−3.3	−10.7	− 8.1
16	− 7.8	+3.2	−1.1	− 0.7	+ 5.7	−6.0	− 7.9	− 3.0
17	− 5.0	+3.2	+0.2	+ 0.4	+ 6.8	−5.4	− 7.6	− 0.4
18	− 2.9	+3.6	+1.2	+ 2.6	+ 6.0	−1.7	− 9.3	− 1.7
19	− 1.2	+4.6	+1.4	+ 3.6	+ 4.0	+2.4	−10.2	− 4.4
20	+ 0.6	+5.2	+0.9	+ 1.9	+ 1.7	+3.8	− 9.1	− 4.4
21	+ 2.4	+4.4	−0.1	− 1.7	− 0.3	+2.5	− 6.7	− 0.7
22	+ 3.5	+1.9	−1.0	− 4.4	− 2.1	+0.9	− 4.9	+ 4.3
23	+ 2.6	−0.9	−1.6	− 3.6	− 3.9	+1.9	− 5.0	+ 6.9

by the application of the corrections derived. For 18 combinations of the series in pairs there are 25 or more stars in common. The results of the comparison are given in Table IX. The first two columns give the series compared and the number of stars in common. The

third and fourth columns show the mean algebraic difference before and after the corrections were applied. In 14 of the 18 series, or in

TABLE VIII

REDUCTION IN MEAN DIFFERENCES PRODUCED BY CORRECTIONS

Series Computed	M	$\bar{\delta}$	ΣM^2	$\Sigma\delta^2$
Alleg.–McC.	3.7	1.5	327.86	54.96
MW 60–Spec.	5.7	1.6	785.87	61.91
Alleg.–Spec.	2.7	1.0	181.90	22.12
McC.–Spec.	2.4	1.2	132.76	35.40
Yerkes–Spec.	6.2	3.5	875.63	280.71
Sproul–Spec.	7.9	2.4	1508.67	138.21
Greenw.–Spec.	4.1	1.8	405.37	74.01
Joh.–Spec.	9.3	2.2	2092.48	117.00
Cape Ph.–Spec.	7.8	4.5	1460.20	494.84

TABLE IX

IMPROVEMENT PRODUCED BY CORRECTIONS

Series Compared	n	Old M	New M	3^h Means			Per Cent New $\Sigma\delta^2$ to Old
Alleg.–McC.	442	−1.8	−0.4	18	0	6	28
McC.–Yerkes	172	−1.8	−0.5	17	0	7	65
Alleg.–Yerkes	133	+0.1	+2.5	12	1	11	89
McC.–Sproul	125	−3.6	+0.8	17	0	7	15
Alleg.–Sproul	113	−5.6	−1.3	16	0	8	63
McC.–Johan.	103	−1.8	+4.2	16	0	8	57
MW 60–McC.	88	−0.4	−3.9	10	0	14	138
Yerkes–Sproul	86	−2.3	0.0	9	1	14	92
MW 60–Alleg.	77	+5.4	+1.1	12	0	12	65
Alleg.–Johan.	72	−7.7	−0.8	20	1	3	25
MW 60–Yerkes	51	+8.3	+5.2	16	2	6	54
McC.–Greenw.	51	+8.0	+5.8	18	1	5	60
Alleg.–Greenw.	49	−1.4	+0.6	7	4	13	205
Johan.–Cape Ph.	47	+1.7	−1.4	17	0	7	77
MW 60–Sproul	41	−2.7	−2.1	12	0	12	90
McC.–Cape Ph.	28	+1.0	+4.4	16	1	7	91
McC.–Van Vleck	28	+13.2	+7.5	14	6	4	54
Yerkes–Johan.	25	−8.5	−1.7	13	6	5	59
Mean	......	......	......	......	......	......	61

78 per cent, there is an improvement. The fifth, sixth, and seventh columns give the number of three-hourly means which are improved,

remain unchanged, and are made less satisfactory, respectively, by the application of the corrections. Again there is an improvement in 78 per cent of the cases. Finally, the last column gives the ratio of the revised to the uncorrected $\Sigma\delta^2$ for the three-hourly means. Here weighted means have been used, since in several cases the numbers of stars involved are very irregular. We find an improvement in 16 of the 18 pairs, or in 89 per cent. In the mean, $\Sigma\delta^2$ is reduced to 61 per cent of its original value. Although the corrections here derived are of course not perfect, there can be little doubt that their application brings the trigonometric parallaxes on to a considerably improved system.

ERRATUM

An error was made in applying the formula in *Astrophysical Journal*, **77**, 194 (*Mt. Wilson Contr.*, No. 463, p. 8), 1933. The formula gives $R=0.275\odot$; the volume is therefore about one-fiftieth that of the sun and the density of the order of several hundred times that of the sun.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
April 1933

PRELIMINARY REPORT ON THE INVESTIGATION OF THE SYSTEM OF H.D. 198287–8[1]

By WILLIAM H. CHRISTIE

ABSTRACT

The discovery of bright hydrogen lines in the spectrum of H.D. 198287–8 in 1921, by Merrill, Humason, and Miss Burwell, led to this investigation of the star. A number of variations in the physical characteristics have been found, on which a preliminary report is given.

Velocity variation.—The star is variable in velocity with a short period of 18.60 days, while the velocity of the system as a whole varies with a period of about 12.3 years. The amplitude of the short-period velocity-curve varies with a period comparable with that of the long-period velocity variation.

Variations in spectrum.—There are two outstanding variations in the spectrum: one in the character and strength of $H\beta$, the other in the intensity of an unidentified line near λ 4232. $H\beta$ consists of a double-emission line, the two components of which vary in both relative and absolute intensity with a period identical with that of the short-period velocity change. λ 4232 appears strong at only one phase; most of the time it is either invisible or else so weak that its identification becomes doubtful.

Variation in light.—The star varies in brightness with a period apparently identical with the short-period velocity variation. The light-curve is similar to that of β Lyrae.

INTRODUCTION

Although the investigation outlined in the present paper is far from complete, it seems advisable to publish now the results already obtained from the study of this remarkable system, since several years must elapse before it can be brought to a close.

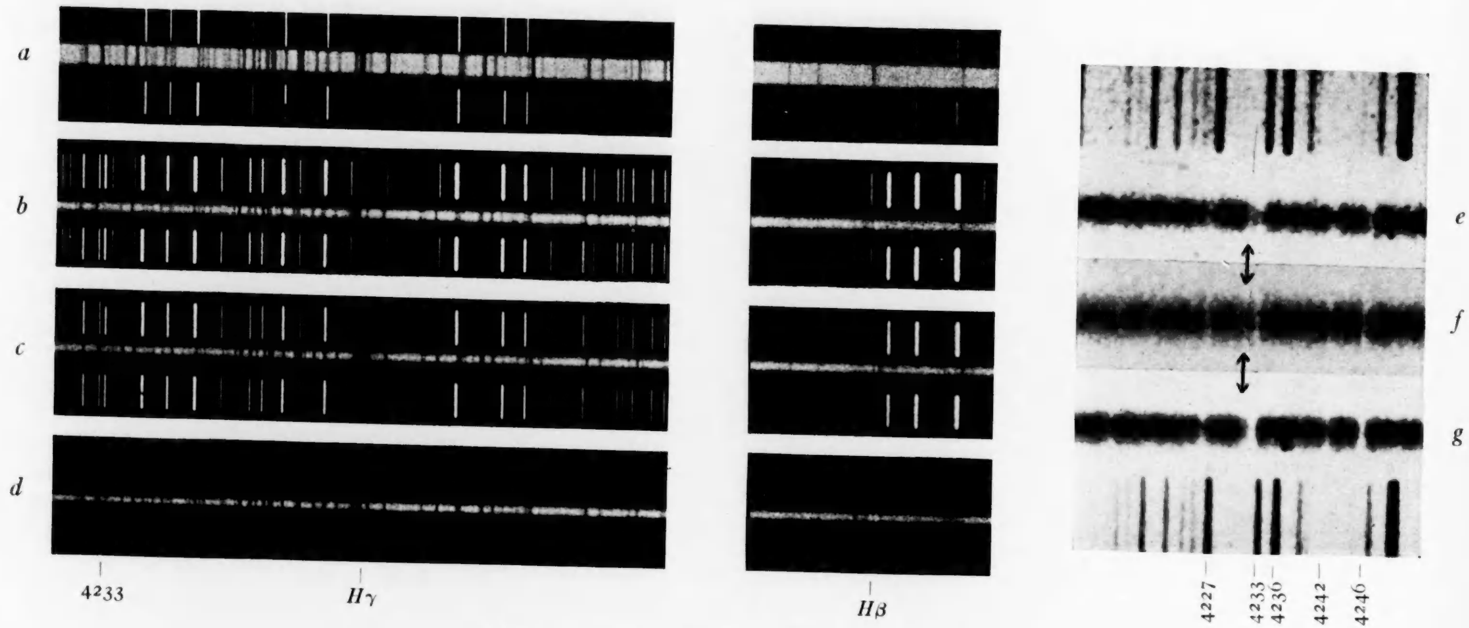
The investigation of H.D. 198287–8 was brought about by the discovery of bright lines in its spectrum by Merrill, Humason, and Miss Burwell, in 1921.[2] This discovery was made with the 10-inch Cooke refractor, equipped with an objective prism, during a search for stars with bright lines. The star was then investigated by Humason with the slit spectrographs attached to the larger instruments, and the radial velocity was found to be variable from measures of the first few plates taken in 1921 and 1922. Observations were continued until 1928, when, owing to the pressure of other work, they were discontinued until the writer took over the investigation in 1929.

The star is classified in the *Henry Draper Catalogue* as F5–A3,

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington,* No. 475.

[2] *Publications of the Astronomical Society of the Pacific,* **35**, 263, 1923.

PLATE VIII



SPECTRA OF ϵ AURIGAE (*a*) AND H.D. 198287–8 (*b–g*)

(*b–d*) H.D. 198287–8 at phases 11.1, 13.5, and 15.5 days, respectively; (*e–g*) enlargements showing variations in spectrum; (*e*) July 29, 1931; (*f*) June 3, 1931; (*g*) July 15, 1932.

with the remark: "The spectrum is composite; Aitken No. 1434; p.a. 256°5; distance 2″30. The bright star must be a close double either visual or spectroscopic, as the spectrum appears to be a blend of two spectra of nearly equal brightness."

The spectrum closely resembles that of ϵ Aurigae, with which it may be compared in Plate I. It consists of numerous strong, sharp lines of ionized iron, titanium, etc., while the neutral lines of these elements are weak. The hydrogen lines are strong and broad, $H\alpha$ and $H\beta$ having emission superimposed upon the underlying absorption, while occasional traces of incipient emission are to be seen at $H\gamma$. A number of lines vary in intensity and character, notably the hydrogen emission lines and a dark line just to the violet of λ 4233 that has not yet been identified. The composite appearance of the spectrum referred to in the note quoted above may be interpreted as a typical characteristic of the spectra of c-stars of this class, and not, at least primarily, as an indication of the presence of a second spectrum; our classification of the spectrum is cF2ev.

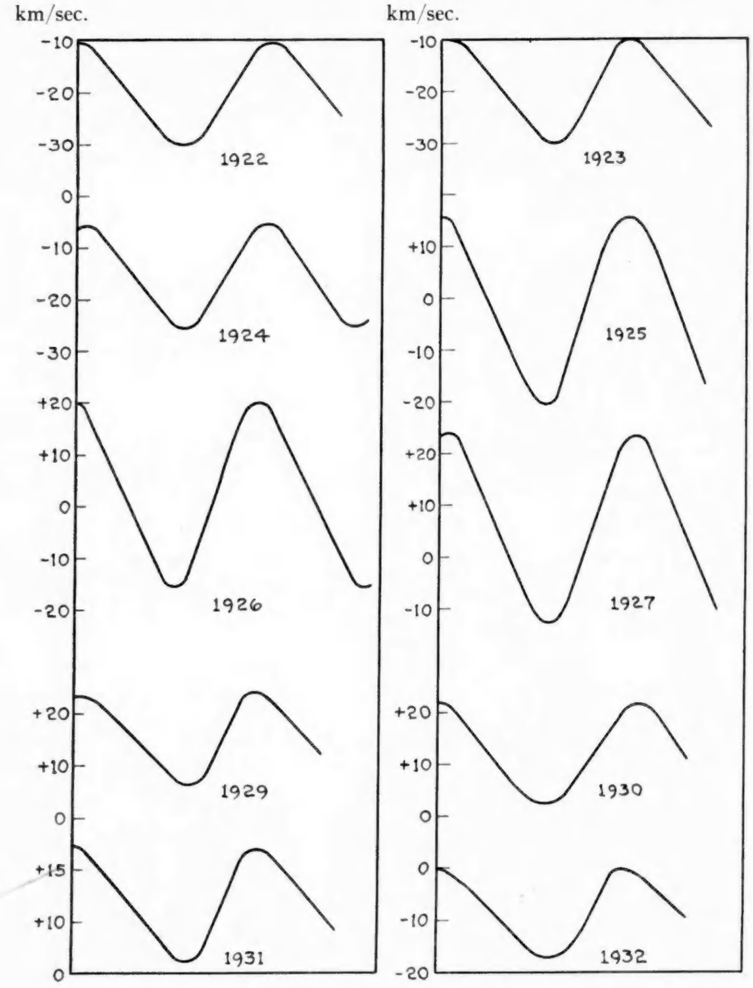
A variation in the light of the star was suspected by Humason and confirmed by the writer.[3] Investigation of this variation has been carried on in conjunction with the spectroscopic studies and will be continued until the work on this star is completed.

VARIATIONS IN RADIAL VELOCITY

Although seventy-six spectrograms had been taken, no periodicity in the velocity variation of this star had been found when the investigation was undertaken by the writer. The general trend of the observations had, however, indicated the presence of a long-period variation of several years. Since the writer at first suspected a short period of less than a day, several plates were taken in succession on three successive nights. No large change in velocity was found during these intervals; thus the possibility of a very short period was eliminated. Many trials were made before the correct period was found, and then it was at once evident why such difficulty had been encountered. Previous close approaches to the period had been masked by variations, both in the amplitude of the velocity-curve and in the velocity of the center of mass of the system. To these

[3] *Ibid.*, **44**, 125, 1932.

variations must be added the unusually large residuals, which greatly increased the difficulties encountered, for although the spectrum is excellent for accurate measurement, the interagreement of



+20

VELOCITY CURVES OF HD 198287-8

FIG. 1

the velocities derived from the individual lines is poor, possibly owing to blends, and the scatter in the observations is unexpectedly large.

The shorter period of velocity change, which for brevity will hereafter be denoted by S.P., is about 18.60 days, with a small uncertainty due to the difficulty of drawing representative curves through the plotted velocities. This period has been derived from the ten velocity-curves obtained since 1921, which extend over two hundred cycles of variation. To avoid duplication, the observational data for these curves will be given in the final publication only, and then with the revised phases resulting from further observations. The individual velocity-curves thus far available are, however, given in Figure 1. In plotting the seasonal observations no correction was applied to the individual velocities for the long-period variation

TABLE I

Year	Velocity of System	Semi-amplitude	No. of Plates
1922	−20.0 km/sec.	10 km/sec.	27
1923	−17.0	9	9
1924	−15.5	11	4
1925	− 3.5	18	8
1926	+ 2.0	20	19
1927	+ 8.5	19	5
1929	+14.0	10	25
1930	+12.0	9	28
1931	+ 3.0	10	23
1932	− 7.5	9	28

(L.P.) This will be done when the orbital elements of L.P. can be determined with sufficient accuracy.

The L.P. variation, that of the center of mass of the system, seems to be about 12.3 years. If this period is correct, the star has made one complete revolution in its orbit since it first came under observation. Only three plates were taken in the first season, however, and these are, of course, insufficient to define the form of the S.P. velocity-curve. During the following season, that of 1922, a sufficient number was obtained, but unfortunately the star was then in that part of the L.P. orbit where it approaches the sun with its maximum velocity, a position ill suited for use in determining the period; so, although the star may now have completed one cycle, the longer period must remain uncertain for at least three years.

Table I gives the velocity of the system and the amplitude of the

S.P. velocity-curves as derived from the individual curves from each season; these two variable quantities are plotted in Figure 2. The provisional elements, derived from the L.P. variation, are given

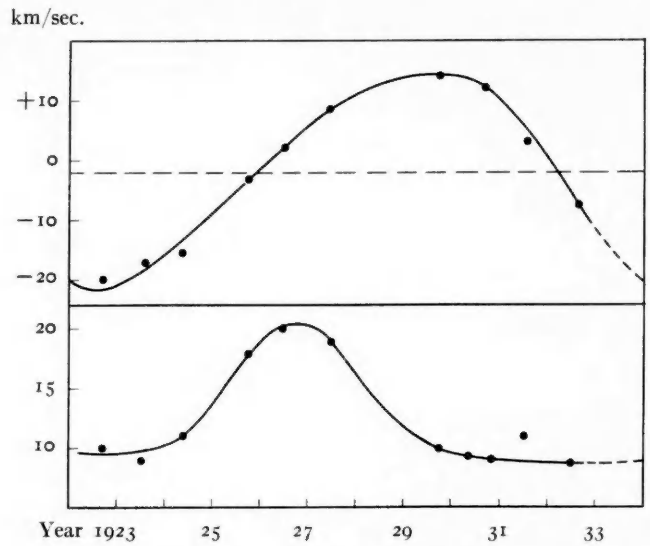


FIG. 2.—Top, variation in velocity of center of mass; bottom, variation in amplitude

TABLE II

PROVISIONAL ELEMENTS OF THE LONG-PERIOD VARIATION OF H.D. 198287–8

$$P = 12.3 \text{ years}$$
$$K = 17.5 \text{ km/sec.}$$
$$\gamma = -2.0 \text{ km/sec.}$$
$$e = 0.2$$
$$\omega = 110^\circ$$
$$T = 1932.5$$
$$a \sin i = 10^9 \text{ km}$$
$$\frac{m_1^3 \sin^3 i}{(m_1+m_2)^2} = 2.3 \odot$$

in Table II. A least-squares solution would undoubtedly improve the provisional elements, but it is questionable whether such a procedure would be worth while at this time.

The peculiar change in the amplitude is well shown by the lower curve in Figure 2. The maximum occurred when the star was at

apastron, the amplitude thereafter decreasing to about one-half its maximum value. The relationship between the amplitude and the position of the star in its orbit is illustrated in Figure 3.

VARIATION IN THE SPECTRUM

The bright hydrogen lines, the discovery of which led to the investigation of this star, vary in character with a period identical with that of the S.P. velocity variation. $H\beta$, the only emission line studied extensively, consists, ordinarily, of broad emission superimposed upon a broader absorption line, while superimposed upon the emission line is a strong narrow absorption line. The red and violet components of the emission vary in intensity and width with the variation in velocity, one component becoming weaker and narrower as the other becomes stronger and broader. This variation may be ascribed to the position of the central and underlying absorption rather than to an intrinsic variation in the emission itself. The ratio of the intensities of the red and violet components of $H\beta$, which will be denoted by *Er* and *Ev*, respectively, have been estimated on practically all the plates measured for radial velocity, with *Er* taken as unity. These estimates, grouped into nineteen normal places according to phase, are plotted in Figure 4. Above the line of equality, i.e., for $Ev > Er$, reciprocals have been taken and Er/Ev plotted, thus making the curve symmetrical about the axis. The similarity of the curves drawn through the points thus plotted and the normal velocity-curve, indicated by the broken line, is remarkable; but this similarity is to be expected if the change in the two components is due to the velocity shift of the absorption line, the emission remaining more or less stationary. Similar changes in intensity are a common and fairly well-known phenomenon found in many Be stars, as Dr.

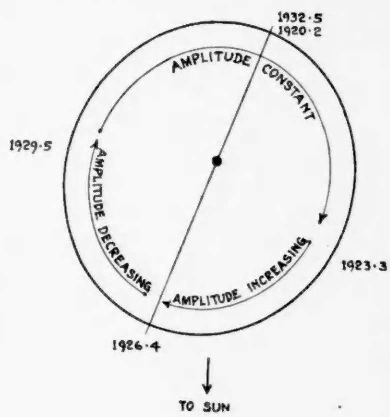


FIG. 3

Merrill pointed out to me some time ago. No measures of the positions of the components of $H\beta$ have been made because few of our spectra are in good focus in this region of the spectrum.

The fact that the absorption appears to move symmetrically with respect to the central emission line, regardless of the motion of the star in the L.P. orbit, indicates that both emission and absorption arise in the atmosphere of the visible system. If the emission did not originate in the atmosphere of this system and did not partake

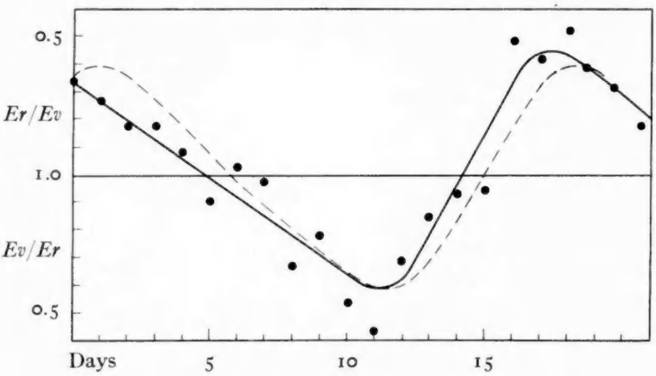


FIG. 4.—Relative intensities of the emission components of $H\beta$

of the long period of velocity variation, we should expect to find the average ratio Ev/Er greater than unity when the star was receding and less than unity when the star was approaching. Actual analysis of the observed data shows an indication of the reverse effect, but no more than may be attributed to errors in estimating the intensities of the two components. Representative contours made from microphotometer tracings of $H\beta$, which show the structure of the line, are given in Figure 5*a*, while Figure 5*b* shows the suggested structure of the combined emission and absorption lines in two characteristic phases. In these hypothetical contours the components due to the emission and absorption have been treated as though they were additive, although the conditions which this assumption involves are probably not actually realized in the star.

The total intensity of the emission is also variable and may be separated into two components: first, a variation due to the velocity shift of the underlying absorption line, which results in an apparent

reduction in the intensity of the line when it is unsymmetrical; and, second, a variation evidently due to the actual reduction of the intensity of the emission, for which no periodicity has yet been found but which is probably connected with the long-period varia-

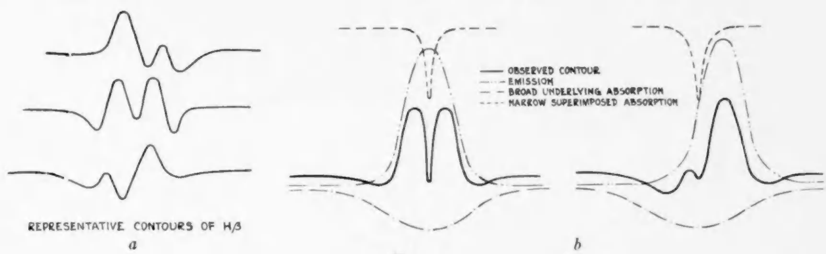


FIG. 5

tion. The first of the two variations of $H\beta$ is shown in Figure 6, where the estimates of the total intensity of the emission have been grouped into nineteen normal places and plotted according to the phase of the S.P. velocity change. No attempt has been made to eliminate the effects of the other variation, hence the large scatter of the observations.

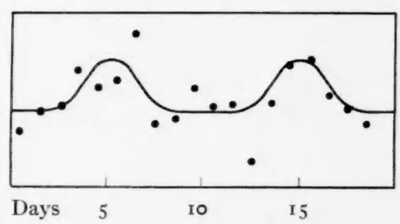


FIG. 6.—Intensity of Hβ emission

One of the most remarkable variations in the spectrum is that of an unidentified line near $\lambda 4232$, which ranges from invisibility to an intensity about one-half that of $H\gamma$. The rise and fall in intensity covers about one-third of the S.P., the maximum occurring when the star has its largest negative velocity. The separation of this line from λ 4233, a strong line due principally to *Fe* II, seems also to vary with the phase. The measured separation is, however, wholly unreliable when the line is weak, because of the existence of numerous very faint lines in this region of the spectrum with which it is likely to be confused. Rejecting all measures for which the identification is uncertain, we find a scatter too great to be attributed to accidental or subjective errors, but with no trace of periodicity. The mean separation of λ 4232 from λ 4233 and the maximum intensity of the line vary from year to year in a manner not yet understood; no cer-

tain trace of the line has been seen on the plates taken between 1922 and 1929. The intensity-curves for λ 4232 for the year's when the line was measurable are given in Figure 7*a*, and typical contours of this region, made from microphotometer tracings, are shown in Figure 7*b*.

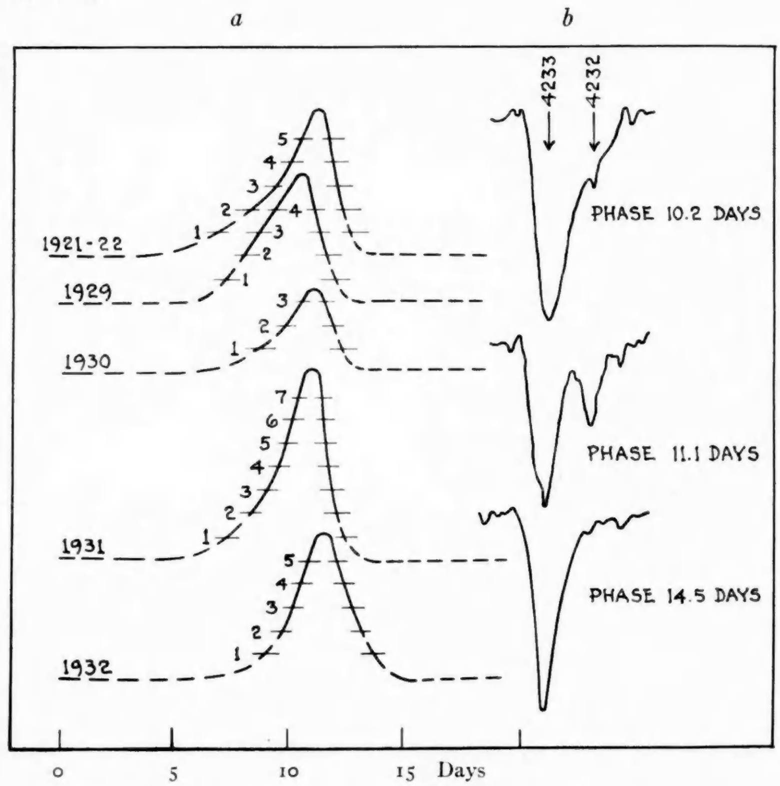


FIG. 7.—(*a*) Intensity of λ 4232; (*b*) typical contours of λλ 4232, 4233

VARIATION IN LIGHT

As already mentioned, Humason suspected this star of being a variable, and when he handed the investigation over to me, he suggested that this possibility be tested. The star was accordingly placed on the program for the new moving-plate camera (*Schraffierkassette*) attached to the 10-inch Cooke refractor. The variation was soon confirmed and has now been followed for nearly two years. Figure 8, which needs little comment, shows the individual determi-

nations of the star's brightness from September 16, 1931, to December 7, 1932, each determination being, as a rule, derived from measures of two plates. A later publication, now in preparation, will give the details of the new instrument with which these results were obtained and the methods used in measuring and reducing the plates. The scale of magnitudes given may need a slight revision, as it depends on only two comparisons with the North Polar Sequence; any change, however, will not materially alter the curve. The phases of the two minima correspond to those of the velocity-curve where it

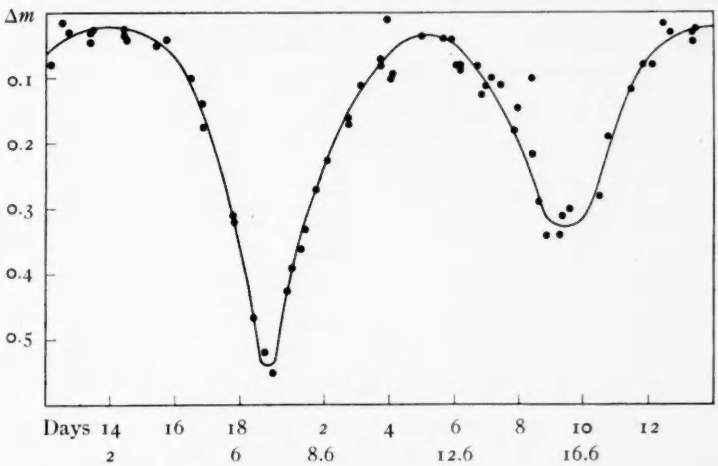


FIG. 8.—Light-curve of H.D. 198287-8. Phases: above, from principal minimum; below, to correspond with other curves.

crosses the γ-axis in the S.P. variation. The lack of symmetry in the light-curve is apparent, the first maximum following the principal minimum being of considerably shorter duration than the following one. Further, the rise to this maximum is much less steep than the preceding fall to minimum, which is additional evidence of the eccentricity found in the velocity-curve.

CONCLUSION

It seems evident from the observations described that we are dealing with a triple system or else with a long-period spectroscopic binary, one member of which simulates a true binary in many ways. The variation of the light of the star in a manner similar to that of

β Lyrae would lead to the conclusion that we are dealing with a very close double, were it not for the evidence against such a hypothesis. The changing amplitude of the S.P. variation without a corresponding increase in period seems to preclude the possibility of the star's being a true binary. The change in amplitude cannot be attributed to precession or nutation when we consider the manner in which the amplitude has been observed to change. The small amplitude of the velocity-curves again precludes the assumption of two bodies unless their orbit is highly inclined to the line of sight, an explanation seemingly ruled out by the character of the observed light-curve. Again, the small amplitude might be attributed to a blending effect of the spectra of the two bodies, were it not for the lack of evidence of a periodic broadening of the spectral lines. Further, the spectrum is characteristic of extremely massive and large stars with extensive and tenuous atmospheres, and when the dimensions of the velocity variations are considered, this seems to preclude still further the possibility of the velocity variation being due to a true binary. Finally, pulsation alone cannot be entertained as an explanation for a light-curve of the type with which we are dealing. Further observations and study along other lines of investigation that have suggested themselves may finally reveal the true nature of these remarkable variations.

In conclusion, I wish to express my indebtedness to Mr. Humason for handing this interesting problem over to me and for the large number of measures that he has placed at my disposal; also to Dr. Merrill for the many helpful discussions from which I have profited, especially in relation to the cause of the variation of $H\beta$. To Mr. Glenn Moore, night assistant at the Observatory, I owe my thanks for a number of the plates taken with the *Schraffierkassette* at important phases when I was unable to obtain them myself.

Carnegie Institution of Washington
Mount Wilson Observatory
May 1933

THE RED TITANIUM OXIDE SYSTEM IN α_1 HERCULIS

By N. T. BOBROVNIKOFF

ABSTRACT

Sixty-six bands belonging to the red system of *TiO* have been measured in the spectrum of α_1 Herculis between λ 6292 and λ 8506. Of these, eleven bands of the sequence -2 have never been observed in the laboratory. The *Q*-heads in the star seem to be much fainter than in the laboratory sources. In addition to the bands in the red system, four bands belonging to the blue-green system of *TiO* and two unclassified bands of *TiO* have been found in the same spectral region in this star.

The *TiO* molecule has three systems of bands. One of the systems recently discovered by Miss F. Lowater[1] is situated in the orange near λ 5600. Only a few heads in this system are known, and the electronic levels cannot as yet be assigned. The blue-green system, transition $^3\Pi - ^3\Pi$, is well known in both the laboratory and celestial sources. The red system, $^3\Sigma - ^3\Pi$, has recently been measured in detail by A. Christy[2] and by Miss Lowater[1] in laboratory sources. It has numerous heads in the red and the near infra-red parts of the spectrum.

The $^3\Pi - ^3\Pi$ system is very prominent in the spectra of the M-type stars. Since the red system has a level in common with the blue-green system, we might expect it to be present in the M-type stars. In fact, Christy identified some bands measured by C. D. Shane[3] in *o* Ceti with the $+2$ sequence of the $^3\Sigma - ^3\Pi$ system. Some of the bands in the M-type stars measured by Merrill[4] and others evidently belong also to this system. Very little, however, has been known of this system as a whole so far as its appearance in celestial spectra is concerned.

Recent progress in manufacturing red and infra-red sensitive plates has made it possible to extend the investigation of celestial spectra into that region. Spectrograms obtained with the autocollimating grating spectrograph of the Yerkes Observatory, used

[1] *Proceedings of the Physical Society*, **41**, 557, 1929.

[2] *Astrophysical Journal*, **70**, 1, 1929.

[3] *Lick Observatory Bulletins*, **10**, 131, 1922.

[4] *Scientific Papers of the Bureau of Standards*, **14**, 487, 1918.

in conjunction with the 69-inch Perkins reflector, show that the most prominent characteristics of the M-type stars in the red and infra-red region are heavy absorption bands. Most of them evidently belong to the red system of *TiO*. The spectrum of α_1 Herculis has been investigated in detail. Only a few bands could not be assigned to *TiO*, and none of these is very prominent.

The dispersion of the spectrograph is about 26.7 A per mm. near λ 6300 and 26.2 near λ 8500. The *Ne* comparison was used and the wave-lengths of the bands were obtained by simple interpolation. Most of the departure from normal dispersion is due to the curvature of the field of the auto-collimating lens. This was somewhat improved by bending the plate in the plateholder, but the residual effect was still considerable. Correction to linear interpolation was found by a graphic construction. By various tests it was found that the wave-lengths of atomic lines could be obtained within 0.1 A, even when interpolation extended to 300 A. The uncertainty of the measures of the bands was, however, considerably more than that, sometimes amounting to 1 A, as some bands, especially *Q*-heads, have no sharp boundary. Furthermore, the measures of the band heads by Christy and by Miss Lowater, made in the laboratory, are not very concordant, sometimes differing by more than 1 A. Therefore, it was thought useless to give the wave-lengths of the bands with precision greater than 0.1 A. In most cases the bands were obviously degraded toward longer wave-lengths, so that the micrometer wire was set on their shorter wave-length edge.

Intensities of the bands were estimated on an arbitrary scale and, of course, are comparable only within a narrow region on each plate, not exceeding, say, 100 A. No precise sensitivity-curves for the plates used are available. In fact, the 1 N Eastman plates used in this work did not behave according to their specifications and are undoubtedly 1 U plates in the present nomenclature of the Eastman laboratory.

Plates used for this investigation are as shown in the accompanying tabulation (see opposite page).

These spectrograms were obtained by Mr. M. Cobb, Miss A. Farnsworth, and myself. All plates except 3 F were treated with

ammonia. The results of measurement are given below. The laboratory wave-lengths are those of Miss Lowater unless otherwise stated. Wave-lengths are given in I.A. For the radial-velocity correction the value $v = -32.6$ km/sec. has been adopted.

Brand	Date	Exposure	Remarks
E.K. 3 F	April 23, 1933	3^h20^m	Sequence +2
1 N	May 19	2 15	+1, 0
1 N	May 31	3 00	+1, 0
1 R	April 24	4 54	−1, −2
1 R	June 9	4 10	−1, −2
1 P	June 11	7 00	Plate somewhat weak

SEQUENCE +2

This sequence has apparently only *R*-heads. It is not intense on the 3 F plate but can be measured with ease. Each head in a triplet

TABLE I

R-HEADS

v', v''	Star		Laboratory		*−Lab.	Remarks
	λ	i	λ	i		
a 2, 0	6292.4	2	6296.0		−3.6	S6294.2
b 2, 0	6322.2	2	6321.7	1	+0.5	S6320.6; *Fe* 6322.7⊙(d4, s6)
c 2, 0	6351.0	3	6350.4		+0.6	Lab. by Christy; S6350.8
a 3, 1	6356.9	3	6357.5		−0.6	S6356.9
b 3, 1	6384.0	3	6382.9	1	+1.1	Christy 6384.4; S6383.1
c 3, 1	6414.7	2	6414.2	1	+0.5	S6413.3
a 4, 2	6420.8	2	6421.9		−1.1	S6419.8; *Fe* 6421.4⊙(d7, s10)
b 4, 2	6447.8	3	6447.8	1	0.0	S6446.9
c 4, 2	6478.1	2	6479.0	1	−0.9	S6477.9
a 5, 3	6484.2	3	6483.6	2	+0.6	
b 5, 3	6512.2	1	6512.6	1	−0.4	
c 5, 3	6543.0	3	6543.8	1	−0.8	S6542.1

is denoted *a*, *b*, *c*, but the intervening atomic lines make the triplet character of the bands not very conspicuous.

In the last column are given the wave-lengths measured by Shane

(denoted S) and strong atomic lines in the solar spectrum with their intensities in the disk (d) and in the spot (s).

The bands $a(2, 0)$ and $a(4, 2)$ have not been observed in the laboratory. Their wave-lengths have been computed according to Miss Lowater's formulae:

$$
\begin{aligned}
v_a &= 14243.7+[864.60(v'+\tfrac{1}{2})-3.765(v'+\tfrac{1}{2})^2] \\
&\quad -[1009.85(v''+\tfrac{1}{2})-5.125(v''+\tfrac{1}{2})^2]\,, \\
v_b &= 14175.5+[866.27(v'+\tfrac{1}{2})-3.81(v'+\tfrac{1}{2})^2] \\
&\quad -[1008.15(v''+\tfrac{1}{2})-4.54(v''+\tfrac{1}{2})^2]\,, \\
v_c &= 14100.7+[867.57(v'+\tfrac{1}{2})-3.98(v'+\tfrac{1}{2})^2] \\
&\quad -[1009.18(v'+\tfrac{1}{2})-4.64(v''+\tfrac{1}{2})^2]\,.
\end{aligned}
$$

In addition to these sixteen bands, the bands due to *TiO* have been measured as shown in Table II.

TABLE II

Star		Laboratory		*−Lab.	Remarks
λ	i	λ	i		
6268.3...	3	6268.3	3	0.0	$aR_a(3, 6)$; V6268.9⊙(d−2N, s4)
6274.3...	1	6275.4	1	−1.1	$\gamma R_a(8, 5)$; V6274.7⊙(d−1, s6)
6334.6...	2	6334.8	1	0.2	Unclassified
6548.5...	3	6551.4	1	−2.8	$aR_c(1, 5)$; S6549.0

Letter a denotes the blue-green system and γ the red system.

SEQUENCE +1

This sequence is very prominent in α_1 Herculis, but is rapidly fading out for the higher transitions of v', v''. The Q-heads are quite faint in comparison with the R-heads, much more so than in the laboratory.

In addition to the bands of this sequence, the bands of *TiO* shown in Table IV have been measured.

TABLE III

v', v''	Star λ	Star i	Laboratory λ	Laboratory i	*−Lab.	Remarks
			R-Heads			
a 1, 0	6650.8	3	6651.5	5	−0.7	S6649.4
b 1, 0	6681.0	4	6681.1	6	−0.1	S6679.0
c 1, 0	6714.6	3	6714.4	6	+0.2	
a 2, 1	6718.1	3	6719.3	6	−1.2	Also $Q_c(1, 0)$; *Ca* 6717.8 ⊙(d5, s8)
b 2, 1	6747.9	2	6747.8	5	+0.1	
c 2, 1	6781.7	6	6781.9	5	−0.2	
a 3, 2	6786.0	6	6785.8	4	+0.2	Also $Q_c(2, 1)$
b 3, 2	6815.0	4	6815.3	5	−0.3	
c 3, 2	6850.4	2	6850.2	6	+0.2	
a 4, 3	6853.1	1	6852.3	6	+0.8	
b 4, 3	....	....	6883.8	3	....	Obliterated by atm. B-group, 6883.8
c 4, 3	6918.9	3	6919.5	4	−0.6	Also $R_a(5, 4)$; atm. 6919.0 *i*9
b 5, 4	6951.9	2	6951.5	....	+0.4	
c 5, 4	6987.7	3	6988.6	1	−0.9	Lab. λ by Christy
			Q-Heads			
a 1, 0	6658.6	1	6757.7	2	−0.9	
a 1, 0	6687.1	1	6686.0	1	+1.1	
b 2, 1	6725.1	1	6724.0	5	+1.1	

TABLE IV

Star λ	Star i	Laboratory λ	Laboratory i	*−Lab.	Remarks
6625.4	2	6626.1	2	−0.7	$aR_a(9, 12)$; *Fe* 6625.0⊙(d0, s3)
6634.8	2	6634.3	2	+0.5	Unclassified; S6634.6
6690.1	1	6691.2	4	−1.1	$aR_b(10, 13)$

SEQUENCE 0

Bands of this sequence are the most prominent feature of the spectrum of the star on N-plates. The remarkable feature of this sequence is the faintness of the *Q*-heads as compared with laboratory sources, and the rapid fading-out of the whole sequence. The bands have quite a complicated structure. Each one of them has two distinct heads, *R* and *Q*, and in addition to this several other subheads. These can be identified also on the reproduction of laboratory spectra, but their significance is not yet clear.

TABLE V

v', v''	Star		Laboratory		*—Lab.	Remarks
	λ	*i*	λ	*i*		
	R-Heads					
a 0, 0	7054.4	10	7054.5	8	−0.1	
b 0, 0	7087.7	10	7087.9	12	− .2	
c 0, 0	7125.7	10	7025.6	14	+ .1	Also $R_a(1, 1)$
b 1, 1	7159.0	2	7159.0	6	0.0	
	Q-Heads					
a 0, 0	7060.1	2	7060.0	....	+0.1	
b 0, 0	7093.1	2	7093.2	....	− .1	
c 0, 0	7132.0	2	7131.3	....	+0.7	Also $Q_a(1, 1)$

SEQUENCE −1

This sequence is represented in the spectrum of α_1 Herculis by numerous well-defined bands. Even though some of them fall into the region of the atmospheric A-band, there is no doubt as to their presence. Especially prominent is the $R_a(0, 1)$ band, neatly separated from the head of the A-band at λ 7593.7.

SEQUENCE −2

This sequence has not been observed in the laboratory, but its occurrence in stellar spectra was predicted by Christy. Its traces can

be seen on the photographs by Miss Lowater. It must be very faint in the laboratory, as atomic lines on the reproduction are quite intense, so that the sensitivity of the plates used by Miss Lowater was

TABLE VI

v', v''	Star		Laboratory		*—Lab.	Remarks
	λ	i	λ	i		
			R-Heads			
a 0, 1	7589.1	10	7589.6	8	−0.5	Lab. Christy 7589.1
b 0, 1	7629.0	9	7628.1	8	+0.9	A-band 7627.0; 7628.2
c 0, 1	7672.6	4	7672.1	10	+0.5	A-band 7670.6; 7671.7; also $Q_a(1, 2)$
a 1, 2	7666.9	8	7666.4	6	+0.5	A-band 7665.9; A+K 7664.9
b 1, 2	7705.6	2	7705.2	8	+0.4	
c 1, 2	7750.5	3	7749.8	4	+0.7	Also $Q_a(2, 3)$
a 2, 3	7743.7	2	7743.2	4	+0.5	
b 2, 3	7783.1	2	7783.4	2	−0.3	
c 2, 3	7828.3	3	7728.0	10	+0.3	Also $Q_a(3, 4)$
a 3, 4	7820.7	2	7820.1	8	+0.6	
b 3, 4	7861.8	1	7861.0	6	+0.8	
c 3, 4	7907.1	2	7907.3	6	−0.2	Also $Q_a(4, 5)$
a 4, 5	7900.3	1	7900.9	4	−0.6	
b 4, 5	7940.1	1	7938.5	.....	+1.6	Lab. λ by Christy
c 4, 5	7987.9	1	7988.1	4	−0.2	
			Q-Heads			
c 0, 1	7679.3	1	7678.4	4	+0.9	
b 1, 2	7713.4	4	7714.6	1	−1.2	*Ni* 7714.3 ⊙(d3, s3)
c 1, 2	7755.8	4	7757.6	1	−1.8	
b 4, 5	7748.6	8	7749.3	1	−0.7	

not rapidly diminishing in this region. In the star the bands of this sequence can be easily measured. The wave-lengths of the bands were computed according to Miss Lowater's (L) and Christy's (Ch) formulae. The latter is

$$\left.\begin{array}{l} v_a = 14172.2 \\ v_b = 14105.8 \\ v_c = 14030.8 \end{array}\right\} + (862.5v' - 3.84v'^2) - (1003.8v'' - 4.61v''^2) .$$

It will be seen that the wave-lengths in α_1 Herculis are in better agreement with Christy's formula.

The somewhat weak P-plate extending to about λ 9100 does not show any prominent band to the infra-red of λ 8506.

TABLE VII

R-Heads

v', v''	Star		Computed		O−C		Remarks
	λ	*i*	L	Ch	L	Ch	
a 0, 2....	8205.5	1	8206.3	8205.8	−0.8	−0.3	
b 0, 2....	8251.5	1	8251.5	8251.1	0.0	+0.4	
c 0, 2....	8301.4	2	8303.6	8302.3	−2.2	−0.9	Atm. 8300.4(*i*3)
a 1, 3....	8289.1	1	8289.1	8288.9	0.0	+0.2	Atm. 8289.5(*i*4)
b 1, 3....	8334.0	1	8335.6	8334.8	−1.6	−0.8	
c 1, 3....	8386.9	2	8388.5	8387.2	−1.6	−0.3	Band on both sides of *Fe* 8387.8 ⊙(d3, s4)
a 2, 4....	8373.4	1	8371.8	8372.6	+1.6	+0.8	
b 2, 4....	8419.7	2	8420.4	8419.5	−0.7	+0.2	
c 2, 4....	8472.8	1	8474.0	8473.0	−1.2	−0.2	
a 3, 5....	8457.1	1	8453.9	8457.0	+3.2	+0.1	
b 3, 5....	8506.6	2	8505.9	8504.7	+0.7	+1.9	

R Lyrae (M5) shows the bands of the +1 sequence as distinctly as does α_1 Herculis. They are quite faint in the spectrum of Antares, and are absent from the spectrum of Arcturus.

Perkins Observatory
June 19, 1933

NOTES

VARIATIONS IN INTENSITIES WITHIN A TRIPLET OF *Si* III IN STELLAR SPECTRA

ABSTRACT

Measures of the relative in[illegible]ities within the *Si* III triplet at λλ 4552, 4567, and 4574 in stellar spectra show de[illegible]ations from the square roots of the emission intensities, in the case of certain stars.

In a study of the *Si* III triplet at λλ 4552, 4567, and 4574 in stellar spectra, O. Struve and C. T. Elvey[1] found that the intensities in absorption were proportional to the square roots of the emission intensities. In that problem the spectra were chosen with respect to narrow, sharp lines and wide, diffuse ("dish-shaped") lines in order to see whether the latter were due to an unusual form of the absorption coefficient. No systematic differences were found between the two types of lines, showing that the widening of the lines was due to a physical cause—rotation of the star. Since that time O. Struve[2] has found that some stars, 17 Leporis in particular, show abnormal intensity relationships within a given multiplet, that is, an enhancement of the strong lines with respect to the weak ones, in the supergiant. It has been confirmed by Struve[3] for the *O* II lines in B-type stars, by Struve and Morgan[4] and by Morgan[5] in stars of types A and F.

It seemed desirable to look into the problem of the *Si* III lines again. Spectrograms of six giants and five dwarfs were obtained with the three-prism Bruce spectrograph, and were analyzed with the microphotometer. Instead of using the largest magnification in making the microphotometric tracing and measuring the complete profile of the absorption line, a smaller scale was employed and the total absorptions were obtained by measuring the average width of the base of the line and its central depth. A comparison of the two

[1] *Astrophysical Journal*, **72**, 267, 1930.

[2] *Ibid.*, **76**, 85, 1932.

[3] *Ibid.*, **74**, 225, 1931.

[4] *Proceedings of the National Academy of Science*, **18**, 590, 1932.

[5] *Astrophysical Journal*, **78**, 158, 1933.

methods was made in the case of 2 β Canis Majoris (using three plates), also of metallic lines in stars of later type, and no systematic errors were found in the range of intensities used in this note.

The results are shown in Table I. The first three columns of measures give the total absorptions or equivalent widths of the lines in angstroms, and the last three columns give the relative intensities

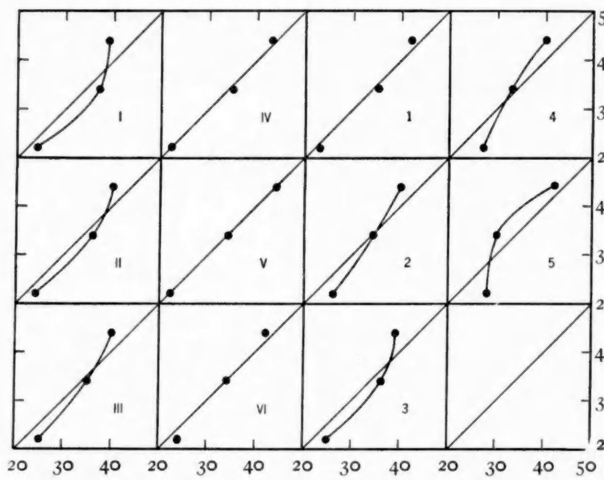


FIG. 1.—Relative intensities of the *Si* III triplet showing deviations from the square-root law. Numbers I–VI are the giants and 1–5 the dwarfs in the table. The ordinates are square roots of emission intensities (sum for triplet equals 100) and the abscissae are the relative absorption intensities (sum equals 100).

adjusted in such a way that the sum equals 100. At the bottom of the table are the square roots of the theoretical intensities and of the emission intensities as measured by Struve and Elvey.[6]

The contents of the table are shown to better advantage in graphical form. In the diagram the relative intensities in absorption are plotted as abscissae, and the square roots of the emission intensities as ordinates. The Roman and Arabic numerals indicate, respectively, the giants and dwarfs as classified by Struve.[7] It was expected that any deviation from the square-root law would show in all members of a group. However, this is not the case, and, instead, it seems to be a property of the individual star. The first two stars in the list

[6] *Loc. cit.*

[7] *Op. cit.*, **78**, 73, 1933.

are decided giants and show marked deviations from the square-root law. The deviation is in the sense that the strongest line of the multiplet is weakened. Three of the giants and one dwarf obey the

TABLE I

INTENSITIES OF *Si* III TRIPLET

Star		No. of Plates	Total Absorption			Relative Intensity		
			4552	4567	4574	4552	4567	4574
	Giants							
I	47 ρ Leo	3	0.47 A	0.44 A	0.29 A	39	37	24
II	44 ζ Per	3	.39	.36	.24	40	36	24
III	21 ϵ CMa	2	.41	.36	.26	40	35	25
IV	53 κ Ori	2	.34	.28	.18	43	35	22
V	2 β CMa	3	.37	.29	.19	44	34	22
VI	24 o^2CMa	1	.15	.12	.08	42	34	24
	Dwarfs							
1	88 γ Peg	2	.13	.11	.07	42	35	23
2	24 γ Ori	1	.14	.12	.09	40	34	26
3	23 τ Sco	2	.14	.13	.09	39	36	25
4	8 β Cep	4	.18	.15	.12	40	33	27
5	17 ζ Cas	3	.14	.10	.09	42	30	28
Theory						45	35	20
Emission						44	34	22

law exceptionally well. One dwarf, 17 ζ Cassiopeiae, shows a large deviation, however, in the sense of the faintest line being too intense. The agreement of the different spectrograms of the same star points definitely to the reality of the deviations. There seems to be no ready explanation for these variations of relative intensities within a multiplet.

C. T. ELVEY

YERKES OBSERVATORY
September 8, 1933

REVIEWS

Lehrbuch der Astronomie. By ELIS STRÖMGREN and BENGT STRÖMGREN. Berlin: Julius Springer 1933. Pp. viii+555. Figs. 186. RM. 30; bound, RM. 32.

The *Lehrbuch* by the two Strömgrens, father and son, is one of the best textbooks on general astronomy now on the market. It covers all branches of astronomy and astrophysics and can be very highly recommended to students and laymen. It can also be recommended to the professional astronomer, whether teacher or investigator, as an excellent reference book, containing a remarkably clear and balanced summary of both old and recent work. The book begins with a brief explanation of astronomical instruments and methods of research; this is followed by an excellent chapter on spherical astronomy and by two chapters on celestial mechanics in which all of the more elementary problems are discussed. One chapter of 59 pages is devoted to the solar system. The concluding chapter, which occupies 172 pages, deals with "Stellar Astronomy and Astrophysics." There is a very useful Appendix giving the derivation of the more important formulae, numerical examples, a table of constants, etc. The text is rather elementary, and the arrangement is such that a layman could enjoy many of the chapters.

The book originated from a revision of an earlier textbook in Danish by Mohn and Geelmuyden. This revision by E. and B. Strömgren appeared in 1931 in the Danish language and has now been enlarged and translated into German.

An interesting feature, which is novel in a book of this character, is the omission of the names of living astronomers, except in a few cases where such names have become inseparably associated with certain astronomical concepts, for example, "Russell-diagram," etc.

The chapters on astrophysics are written from a theoretical, rather than an observational, point of view. Many readers will welcome this method of approach, although occasionally the authors are forced to introduce formulae without proof or without a very clear explanation of the principles underlying them. It is doubtful, for instance, whether the beginner or the layman would be able to follow the explanation of the broadening of absorption lines by radiation damping. The derivation of

the formulae for natural line width is, of course, too complicated for an elementary textbook, but its omission must leave a feeling of disturbance in the mind of a painstaking student who is in the habit of acquiring a complete understanding of all steps leading to a certain result. The more advanced student who has a good background in physics will have no such difficulty. For him the chapters on astrophysics will be a real inspiration.

O. Struve

The Universe in the Light of Modern Physics. By Max Planck. New York: W. W. Norton & Co., 1931. Pp. 114. $2.00.

This book, which came from the press in 1931, is of great interest because it expresses the point of view of a man who, with Einstein and Lorentz, laid the foundation of modern physics. It is written for those who find their world of determinism somewhat uncomfortable, as well as for those who forget that indeterminism is not without its perils. Dr. Planck puts the argument for determinism bluntly when he says: ". . . . In my opinion, so long as any choice remains, determinism is in all circumstances preferable to indeterminism simply because a definite answer to a question is always preferable to an indefinite one." The book closes with the statement that "modern physics impresses us particularly with the truth of the old doctrine which teaches that there are realities existing apart from our sense-perceptions, and that there are problems and conflicts where these realities are of greater value for us than the richest treasures of the world of experience." This is an assertion so burdened with mystical meaning that "lesser breeds without the law" would do well to ponder it before they give themselves wholly to the gods of mensuration.

C. C. Crump

Greek Astronomy. By Sir Thomas L. Heath. ("The Library of Greek Thought," ed. Ernest Barker.) London: J. M. Dent & Sons, Ltd.; New York: E. P. Dutton & Co., Inc., 1932. 8vo. Pp. lvii+192. Cloth, $1.75.

"The Library of Greek Thought"—to which the book under review belongs as the unnumbered tenth volume—presents selected source material translated into English and arranged according to subjects: historical thought, social life, economics, medicine, literary criticism, etc.

The relation of Sir Thomas Heath's *Greek Astronomy* to his *Aristarchus of Samos* (1913) is similar to that of *A Manual of Greek Mathematics* (1931)

to *A History of Greek Mathematics* (1921); it is considerably shorter and ought to interest a wider circle of readers. A rather elaborate history of Greek astronomy to Aristarchus formed the bulk of the 1913 volume, intended, originally, to give a detailed study of "the ancient Copernicus." *Greek Astronomy* offers a general view of the whole field; the introductory chapter is rather short—forty-five pages—but it succeeds in clearly outlining the development of Greek astronomy; the essential part of the book consists of selected extracts—Greek thought translated into English.

The fragments from Thales, Anaximander, Anaximenes, Pythagoras, Alcmaeon, Xenophanes, Heraclitus, Parmenides, Empedocles, Anaxagoras, the Pythagoreans, Leucippus, and Democritus, as preserved by later writers, offer, of course, the highest number of profound passages per page; they are followed by extracts from Plato, Eudoxus and Callippus, Aristotle, and Heraclides of Pontus. Most of the translations have been made by Heath, and many of them were included in *Aristarchus of Somos*. Extracts from Euclid, Aristarchus, Eratosthenes, Aratus, Posidonius, Geminus, Hipparchus, Ptolemy, Strabo, Cleomedes, Plutarch, and from the Aristotelian corpus, form, in some respects, an anticlimax; we find the same perfect style of translation and about the same amount of reading matter—most of it not included in the 1913 volume—but the number of brilliant thoughts per page is noticeably lower. A chart of the constellations of the Northern Hemisphere and a good Index conclude the volume.

A Source Book in Astronomy, by H. Shapley and H. E. Howarth, covers the period from Copernicus to George Darwin. The gap from the end of antiquity to the end of the Middle Ages remains to be filled.

A. Pogo

The Book of the Sky. By Matthew Luckiesh. 2d ed. New York: E. P. Dutton & Co., 1933. Pp. xii+335. $3.00.

This is an enlarged edition of an interesting book describing the beauties of the daytime sky, the clouds and their associations: wind, lightning, and weather. The first edition of this book was a pioneering effort to draw the attention of the laity to the phenomena to be seen in the daytime sky. Much of its contents has been drawn from the author's experiences in flying during the World War. The second edition of this book has been enlarged to contain several additional chapters of interest to the aerial traveler: "The Ways of the Winds," "Whence Weather," "To Know Weather," "Lightning," "Bumpiness," "Measurements of the Weather," and "The Sky Vault."

C. T. Elvey